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# Analysis, PDEs & Applied Mathematics
## – List of abstracts –

### *On the Navier problem for the anisotropic Stokes equations and for the anisotropic Navier-Stokes equations*

**Andrei-Florin Albişoru**

Babes-Bolyai University

Abstract: Our purpose is to provide well-posedness results for a Navier type problem for the anisotropic Stokes system with $L^\infty$ viscosity coefficient tensor satisfying a certain ellipticity condition in the setting of bounded Lipschitz domains of $\mathbb{R}^3$. This can be achieved by using a variational approach. We also establish an existence result for the Navier problem for the anisotropic Navier-Stokes system in bounded Lipschitz domains in $\mathbb{R}^3$. Joint work with M. Kohr, I. Papuc and W.L. Wendland.

### *An extension operator of Roper-Suffridge and Graham-Kohr type*

**Mihai Aron**

Babes-Bolyai University

Abstract: We generalize the Roper-Suffridge-Graham-Kohr extension operator,

$$f \mapsto \Psi_{n,\alpha,\beta},$$

where $f$ is normalized locally univalent function on the unit disc of the complex plane and

$$\Psi_{n,\alpha,\beta}(f)(z) = \left( f(z_1), \left[ \frac{f(z_1)}{z_1} \right]^\alpha \left[ f'(z_1) \right]^\beta \tilde{z}, \right),$$

where $z = (z_1, \tilde{z})$ belongs to the unit ball of $\mathbb{C}^n$. We address to the problem of preserving starlikeness and convexity through the new operator. The newly introduced operator provides a new method for constructing starlike functions on the unit ball of $\mathbb{C}^n$. Also, we improve the result regarding preservation of starlikness in case of extension operator $\Psi_{n,\alpha,\beta}$. Finally, we propose an open problem and present a brief introductory study on it.

## *Control strategies for the stabilization of a predator population in a reaction-diffusion system*

**Teodora Baciu**

Alexandru Ioan Cuza University

Abstract: This study explores the problem of eradicating an invasive predator population within a seasonally fluctuating environment - an issue of growing importance in ecological management. We investigate this challenge using a general prey-predator model that features nonlocal terms, local diffusion, and time-periodic parameters, offering a realistic representation of the systems complex behavior. This work shows that long-term eradication of a predator population can be achieved by applying targeted control strategies-either directly to the predators or indirectly through their prey-even if the intervention is limited to just a portion of the habitat, rather than the entire area. We derive rigorous conditions for successful eradication, based on the sign of the principal eigenvalue associated to a non-self-adjoint parabolic operator. The results enhance our understanding of population dynamics in reaction diffusion systems and lay the groundwork for more sophisticated and ecologically sound strategies for species control.

## *Regularity of elliptic equations with singular coefficients*

**Ana Maria Adelina Călina**

University of Bucharest & IMAR

Abstract: We present some partial regularity results for solutions of elliptic equation with singular coefficients of the form $-\Delta v + \lambda x \cdot \nabla v/|x|2 = f$. The aim of this study is to analyze the influence of the unbounded perturbation term like $\lambda x \cdot \nabla v/|x|2$ in the regularity behavior of the solutions of elliptic problems. In this study the regularity depends in a deep way on the value of the spectral parameter $\lambda$. This talk is based on the joint work with Cristian Cazacu. Partially supported by the doctoral fellowship from the University of Bucharest and the NSF-UEFISCDI grant ROSUA-2024-0001 at IMAR.

## *Homogenisation of integro-differential equations*

**Adina Ciomaga**

Octav Mayer Institute of Mathematics of the Romanian Academy & Université de Paris

Abstract: I will talk about recent developments in the theory of homogenisation for nonlocal Hamilton-Jacobi equations, associated with Levy-Itô integro-differential operators. A typical equation is the fractional diffusion coupled with a transport term, where the diffusion is only weakly elliptical. Homogenization is established in two steps: (i) the resolution of a cellular problem - where Lipshitz regularity of the corrector plays a key role and (ii) the convergence of the oscillating solutions towards an averaged profile - where comparison principles are involved. I will discuss the interplay of the regularity of solutions and comparison principles for nonlocal equations, and the difficulties we face when compared with local PDEs.

## *Invariance theory in the space of probability measures and mean-field dynamics*

**Alberto Corella**

Technische Universität Wien

Abstract: This presentation is meant to discuss the invariance theory of dynamical systems in the mean-field limit. It focuses on continuity equations in the space of probability measures, viewed as a pseudo-Riemannian manifold with a metric tensor based on optimal transport. In this context, a set is invariant if, whenever an initial condition belongs to it, the corresponding solution of the continuity equation remains inside it.

## *Bourgain-Brezis type solutions via real interpolation*

**Eduard Curcă**

University of Warsaw

Abstract: In 2002 Bourgain and Brezis proved that given a vector field $v \in \mathcal{S}' \cap \dot{W}^{1,d}$ (on $\mathbb{R}^d$ with $d \geq 2$) there exists a vector field $u \in L^\infty \cap \dot{W}^{1,d}$ such that

$$\operatorname{div} u = \operatorname{div} v. \qquad (*)$$

We discuss a similar result where the space $\dot{W}^{1,d}$ is replaced by the real interpolation space

$$X_{\theta,q} := (L^\infty, \dot{H}^{d/2})_{\theta,q},$$

where $\theta \in (0,1)$, $q \in [1,\infty]$. More precisely, we show that given a vector field $v \in \mathcal{S}' \cap X_{\theta,q}$ there exists a vector field $u \in L^\infty \cap X_{\theta,q}$ satisfying $(*)$.

Due to the fact that we do not have yet a "direct" description of the space $X_{\theta,q}$, this work is still in progress.

## *Finite Element Approximation of the Fractional Sobolev Constant*

**Andreea Dima**

IMAR

Abstract: In the study of eigenvalue problems, for Finite Element Approximation of the Fractional Sobolev Constant, I focused on the analysis of the fractional Laplacian operator.

For the estimation, I first proved that this constant on $\mathbb{R}^N$ is the same as that on a domain.

Finally, I hope to obtain an order of magnitude related to the difference between this constant and its finite approximation.

## *Quantitative results on the subgradient extragradient method*

**Nicoleta Dumitru**

University of Bucharest

Abstract: An essential problem in variational analysis and optimization is the Variational Inequality Problem (VIP), which consists in finding a point $u \in C$ such that

$$\langle f(u), x-u\rangle \geq 0 \quad \text{for all } x \in C,$$

where $C$ is a nonempty, closed and convex subset of a Hilbert space $H$, and $f : H \to H$.

The extragradient method (EGM), introduced by Korpelevich in 1976, is a widely used iterative scheme for solving the VIP in Euclidean spaces. Recently, Pischke applied proof mining techniques to compute, for the first time, uniform rates of metastability for EGM. Furthermore, rates of convergence were obtained under additional metric regularity assumptions.

The extragradient algorithm requires two projections onto $C$ per iteration, which can be computationally expensive. To address this, Censor, Gibali, and Reich introduced the subgradient extragradient method (SEGM), which replaces the second projection with one onto a subgradient half-space, making it more efficient. SEGM was initially developed for Euclidean spaces and later extended to Hilbert spaces, along with further modifications.

In this talk, we discuss work in progress that consists in adapting Pischke's quantitative analysis of Korpelevich's extragradient algorithm to the subgradient extragradient method. This is joint work with Laurenţiu Leuştean.

## *Physics-informed neural networks for personalized hemodynamics modeling*

**Andrei Gasparovici**

Babes-Bolyai University

Abstract: The ability to model organ-specific geometries and predict various quantities in silico can advance personalized patient care through implant simulation or treatment outcome prediction. We present a two-stage, geometry-aware deep learning pipeline that solves the steady incompressible Navier-Stokes equations in patient-specific aortas, enabling in-silico blood pressure and velocity predictions under different simulated medical conditions.

We begin by representing each patients aortic shape mesh as the zero-level set of a neural distance field conditioned by a learnable embedding vector. This vector provides a compact, low-dimensional representation of the geometry, making it useful for distinct applications such as synthetic shape generation.

Next, we leverage the latent vectors to condition a physics-informed neural network, trained to predict the peak-systole velocity and pressure fields. Preliminary results demonstrate good generalizability of the embeddings, enabling the generation of realistic synthetic shapes and accurate predictions for pressure and velocity, with only a small performance gap in pressure. We hypothesize this gap is due to the inherently high variability of pressure and explore potential strategies for addressing it.

Joint work with Alex Şerban (Foundational Technologies, Siemens SRL).

## *Solvability of discounted global Eikonal equations*

**Trí Minh Lê**

Technische Universität Wien

Abstract: Eikonal equations in metric spaces have strong connections with the local slope operator (or the De Giorgi slope). In this talk, based on our current work [1], we discuss an analogous equation based on the global slope operator $G$, expressed as $\lambda u + G[u] = \ell$, where $\lambda \geq 0$. As a consequence of our results, we establish a new representation formula for lower semicontinuous functions in terms of the global slopes.

[1] T. M. Lê and S. Tapia-Garca, On (discounted) global Eikonal equations in metric spaces, https://doi.org/10.48550/arXiv.2410.00530.

## *Mesh-free method for Dirichlet eigenpairs of the Laplacian with potential on star-shaped domains*

**Dragoş Manea**

IMAR

Abstract: This project addresses the numerical approximation of $L^2$ Dirichlet eigenpairs for the Laplace operator $\Delta$ with a radial potential $V$ on a general two-dimensional star-shaped domain $\Omega$.

We propose a mesh-free computational method inspired by the Method of Particular Solutions (MPS) employed for the case of the Laplacian (i.e., $V = 0$). A key challenge in extending this method to a more general potential $V$ is the absence of Bessel function equivalents for the operator $-\Delta+V$. To address this, we first solve the eigenvalue problem on a well-behaved domain - a ball enclosing $\Omega$ - without boundary conditions. For each $\tilde{\lambda} \in \mathbb{R}$, we express the eigenvalue problem $-\Delta u + Vu = \tilde{\lambda}u$ in polar coordinates and, by using Fourier series with respect the angle $\theta$, we obtain a system of ODEs. Numerically solving these ODEs using one-dimensional FEM yields a basis of eigenfunctions for $\tilde{\lambda}$ on the ball, from which we select a finite subset for the numerical analysis. One major advantage is that this basis does not depend on $\Omega$, so one can analyse multiple domains simultaneously at a low computational cost.

The final step involves a Generalised SVD optimisation technique to minimise function values on the boundary of $\Omega$ within the finite-dimensional space formed in the previous step. If the minimised value on the boundary is sufficiently small, then $\tilde{\lambda}$ is close to an actual Dirichlet eigenvalue $\lambda$ of $-\Delta + V$.

## *A bridge between Random Matrix Theory and Schramm-Loewner Evolutions Theory*

**Vlad Margarint**

University of North Carolina at Charlotte

Abstract: I will describe a newly introduced toolbox that connects two areas of Probability Theory: Schramm-Loewner Evolutions and Random Matrix Theory. This machinery opens new avenues of research that allow the use of techniques from one field to another. One aspect of this research direction is centered in an interacting particle systems model, namely the Dyson Brownian motion. I will first describe the toolbox and then I will describe one of the recent applications. Then, I am to describe some of the interesting open problems that emerge using this newly introduced toolbox. This is a joint work with A. Campbell and K. Luh.

## *A general quasilinear elliptic problem with variable exponents and Neumann boundary conditions for image processing*

**Bogdan Maxim**

University of Craiova

Abstract: The aim of this talk is to present existence and uniqueness results for a general quasilinear elliptic problem with homogeneous Neumann boundary conditions, often associated with image processing tasks like denoising. The novelty is that we surpass the lack of coercivity of the Euler-Lagrange functional with an innovative technique that has at its core the idea of showing that the minimum of the energy functional over a subset of the space $W^{1,p(x)}(\Omega)$ coincides with the global minimum. The obtained existence result applies to multiple-phase elliptic problems under remarkably weak assumptions.

## *A time-based direct-segment air traffic trajectory model*

**Radu-Adrian Mihai**

National University of Science and Technology Politehnica Bucharest

Abstract: We propose a practical method for optimizing aircraft trajectory and sequencing in highly constrained airspaces with mixed Visual and Instrument Flight Rules (VFRIFR) traffic. Rather than targeting optimal solutions for a set of standard routes or a free-route scenario (which allows any possible trajectory), our approach focuses on generating operationally feasible trajectories aligned with real ATC practices. Specifically, VFR flights which cannot be vectored are modeled as direct segments toward predefined geographic reference points. Each trajectory is defined by the timing and duration of these segments. This results in a simplified model that yields realistic, controller-friendly solutions for complex terminal environments.

## *A Hardy-Hénon equation with non-linearity*

**Radu Ordean**

University of Bucharest

Abstract: We present existence and nonexistence results for a class of nonlinear elliptic equations of Hardy-Hénon type involving singular potentials. Specifically, we study the

equation $-\Delta u - |x|^\sigma u^p + u^q = 0$ on $\mathbb{R}^N$, under suitable assumptions on the parameters $p$, $q$, and $\sigma$. The solutions are constructed using the Direct Method of Variational Calculus and are further shown to be radially symmetric and compactly supported. Also, they minimize a certain Caffarelli-Kohn-Nirenberg inequality. Nonexistence results for $\sigma \leq 2$ are obtained via a Pohozaev-type identity. Furthermore, these results are extended to equations of the form $-\Delta u - K(x)u^p + u^q = 0$, where the weight function $K$ satisfies appropriate regularity conditions.

This is a work in progress in collaboration with Cristian Cazacu (University of Bucharest & ISMMA, Romanian Academy). Partially supported by a doctoral fellowship from the University of Bucharest.

## *Hypercontractivity of the Heat Flow on* $\mathsf{RCD}(0, N)$ *Spaces*

**Alexandru Pîrvuceanu**

Babes-Bolyai University

Abstract: Using the sharp $L^2$-logarithmic Sobolev inequality, we prove sharp hypercontractivity bounds of the heat flow $(\mathsf{H}_t)_{t\geq 0}$ on $\mathsf{RCD}(0, N)$ spaces. If equality holds in this sharp estimate for a prescribed time $t_0 > 0$ and a nonzero nonnegative extremiser $f$, it turns out that the $\mathsf{RCD}(0, N)$ space has an $N$-Euclidean cone structure and $\mathsf{H}_{t_0} f$ is a Gaussian whose dilation factor is reciprocal to $t_0$ up to a multiplicative constant. On complete Riemannian manifolds with nonnegative Ricci curvature we provide a full characterisation of the equality case. This is joint work with Alexandru Kristály.

## *Sharp second order inequalities with distance function to the boundary*

**Teodor Ruginǎ**

University of Bucharest & ISMMA

Abstract: In this poster, we present generalizations to the $L^p$-setting, $p > 1$, of the Hardy-Rellich inequalities on domains of $\mathbb{R}^N$ with singularity given by the distance function to the boundary. The inequalities we obtain are either sharp or give a new bound for the sharp constant, while also depending on the geometric properties of the domain and its boundary. We also present some applications to the existence and non-existence of non-trivial solutions for a singular problem involving the $p$-Bilaplacian operator.

## *On a Rayleigh-type quotient involving a variable exponent which depends on test functions*

**Anisia Teca**

University of Craiova

Abstract: In this talk we discuss the positivity of the infimum for a Rayleigh-type quotient involving a variable exponent which depends on test functions. In particular, we give some sufficient conditions on these functions in order to get the positivity of the infimum and we provide examples of test functions for which the infimum vanishes. This is a joint work with Mihai Mihăilescu and Denisa Stancu-Dumitru.