Regula lui Leibniz pentru derivarea sub integrală

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Cuprins

Regula de integrare Leibniz a fost introdusă de către matematicianul Fredrick S. Woods în cartea "Advanced calculus".Ulterior,a fost promovată de fizicianul Richard Feynman care a considerat-o in bibliografia sa ca fiind "arma lui secretă".Din acest motiv,metoda este foarte des intalnită sub numele de "procedeul de integrare Feynman."

Pasul I

$$\int_0^1 \frac{x^2-1}{\ln x}dx = ?$$

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$$I : \mathbb{R}\to\mathbb{R};\, I(a) = \int_0^1 f(x,a)dx$$

Pasul II

$$I(0) = \int_0^1 f(x, 0)dx$$

Pasul II

$$I(0) = \int_0^1 f(x,0)dx$$

$$I(0) = \frac{x^0 - 1}{\ln x} = 0$$

Pasul III

$$I'(a) = \int_0^1 \frac{\partial f}{\partial a}(x, a)dx$$

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$$I'(a) = \int_0^1 x^a dx = \frac{1}{a+1}$$

Pasul IV

$$I(a) = \int \frac{1}{a+1}da = \ln(a+1) + C$$

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$$I(0) = C \to C = 0$$

Pasul V

$$I(2) = \ln 3$$

$$\int_{-\infty}^{\infty} \frac{\sin x}{x} dx = ?$$

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$$f : \mathbb{R} \times [0,\infty] \to \mathbb{R}; f(x,a) = \frac{e^{-ax}\sin x}{x}$$

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Definim

$$J = \int e^{-ax}\sin x dx$$

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$$I'(a) = -\int_0^\infty e^{-ax}\sin x dx$$

Definim

$$J = \int e^{-ax}\sin x dx$$

Deci

$$J = \int e^{-ax}(\cos x)' dx$$

$$J = -e^{-ax}\cos x + \int -ae^{-ax}\cos x dx$$

$$J = -e^{-ax}\cos x + \int -ae^{-ax}\cos x dx$$

$$J = -e^{-ax}\cos x - ae^{-ax}\sin x + a\int -ae^{-ax}\sin x dx$$

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$$J = -e^{-ax}(\cos x + a\sin x) - a^2 J$$

Deci

$$(1+a^2)J = -e^{-ax}(\cos x + a\sin x)$$

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$$J = -\frac{e^{-ax}(\cos x + a\sin x)}{1+a^2}$$

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Atunci

$$I'(a) = J\Big|_0^{\infty} = 0 - \frac{1}{1+a^2} = -\frac{1}{1+a^2}$$

Deci

$$(1+a^2)J = -e^{-ax}(\cos x + a\sin x)$$

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Atunci

$$I'(a) = J\Big|_0^{\infty} = 0 - \frac{1}{1+a^2} = -\frac{1}{1+a^2}$$

Vom integra după a

$$I(a) = -\int \frac{1}{1+a^2}da = -arctg(a) + C$$

$$I(0) = C \Rightarrow \int_0^{\infty} \frac{\sin x}{x}dx = C$$

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Dar

$$I(0) = \frac{\pi}{2}$$

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Dar

$$I(0) = \frac{\pi}{2}$$

Aşadar

$$\int_{-\infty}^{\infty} \frac{\sin x}{x}dx = \frac{2\pi}{2} = \pi$$

$$\int_0^{\frac{\pi}{2}} \frac{\ln(\cos\theta)}{1+\sin^2\theta}d\theta = ?$$

$$\int_0^{\frac{\pi}{2}} \frac{\ln(\cos\theta)}{1+\sin^2\theta}d\theta = ?$$

Folosim substituţiile

$$\cos\theta = \frac{1}{\sqrt{1+tg^2\theta}}$$

$$\sin\theta = \frac{tg\theta}{\sqrt{1+tg^2\theta}}$$

$$\int_0^{\frac{\pi}{2}} \frac{\ln(\cos\theta)}{1+\sin^2\theta} d\theta = ?$$

Folosim substituţiile

$$\cos\theta = \frac{1}{\sqrt{1+tg^2\theta}}$$

$$\sin\theta = \frac{tg\theta}{\sqrt{1+tg^2\theta}}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\ln\frac{1}{\sqrt{1+tg^2\theta}}}{1+\frac{tg^2\theta}{1+tg^2\theta}} d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{\ln(\cos\theta)}{1+\sin^2\theta}d\theta = ?$$

Folosim substituţiile

$$\cos\theta = \frac{1}{\sqrt{1+tg^2\theta}}$$

$$\sin\theta = \frac{tg\theta}{\sqrt{1+tg^2\theta}}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\ln\frac{1}{\sqrt{1+tg^2\theta}}}{1+\frac{tg^2\theta}{1+tg^2\theta}}d\theta$$

$$I = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\ln(1+tg^2\theta)}{1+2tg^2\theta}(1+tg^2\theta)d\theta$$

Notăm

$$tg\theta = t \Rightarrow \frac{1}{cos^2\theta}d\theta = dt$$

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Definim

$$f : \mathbb{R} \times [0, \infty] \to \mathbb{R}; f(t, \alpha) = \frac{\ln(1+\alpha^2)}{1+2t^2}$$

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$$f : \mathbb{R} \times [0,\infty] \to \mathbb{R}; f(t,\alpha) = \frac{\ln(1+\alpha^2)}{1+2t^2}$$

$$I : \mathbb{R} \to \mathbb{R}; I(\alpha) = \int_0^{\infty} f(t,\alpha)dt$$

$$I(0) = \int_0^{\infty} \frac{\ln 1}{1+2t^2} dt = 0$$

$$I(0) = \int_0^\infty \frac{\ln 1}{1+2t^2} dt = 0$$

$$I'(\alpha) = \int_0^\infty \frac{\partial f}{\partial t}(t,\alpha)$$

$$I(0) = \int_0^{\infty} \frac{\ln 1}{1+2t^2} dt = 0$$

$$I'(\alpha) = \int_0^{\infty} \frac{\partial f}{\partial t}(t, \alpha)$$

Deci

$$I'(\alpha) = \int_0^{\infty} \frac{t^2}{(1+2t^2)(1+\alpha t^2)} dt$$

Aşadar

$$I'(\alpha) = \frac{-1}{2-\alpha}\int_0^{\infty}\frac{dt}{1+2t^2} + \frac{1}{2-\alpha}\int_0^{\infty}\frac{dt}{1+\alpha t^2}$$

Aşadar

$$I'(\alpha) = \frac{-1}{2-\alpha}\int_0^\infty \frac{dt}{1+2t^2} + \frac{1}{2-\alpha}\int_0^\infty \frac{dt}{1+\alpha t^2}$$

$$I'(\alpha) = \frac{-1}{2-\alpha}\frac{1}{\sqrt{2}} arctgx\sqrt{2}\Big|_0^\infty + \frac{1}{2-\alpha}\frac{1}{\sqrt{2}} arctgx\sqrt{\alpha}\Big|_o^\infty$$

Aşadar

$$I'(\alpha) = \frac{-1}{2-\alpha}\int_0^{\infty}\frac{dt}{1+2t^2} + \frac{1}{2-\alpha}\int_0^{\infty}\frac{dt}{1+\alpha t^2}$$

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$$I'(\alpha) = \frac{\pi}{2(2-\alpha)}\left(\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{\alpha}}\right)$$

Deci

$$I(\alpha) = \frac{-\pi}{2\sqrt{2}} \int \frac{d\alpha}{2-\alpha} + \frac{\pi}{2} \int \frac{d\alpha}{\sqrt{\alpha}(2-\sqrt{\alpha})}$$

Deci

$$I(\alpha) = \frac{-\pi}{2\sqrt{2}} \int \frac{d\alpha}{2-\alpha} + \frac{\pi}{2} \int \frac{d\alpha}{\sqrt{\alpha}(2-\sqrt{\alpha})}$$

Notăm

$$J(\alpha) = \frac{\pi}{2} \int \frac{d\alpha}{\sqrt{\alpha}(2-\sqrt{\alpha})}$$

Deci

$$I(\alpha) = \frac{-\pi}{2\sqrt{2}} \int \frac{d\alpha}{2-\alpha} + \frac{\pi}{2} \int \frac{d\alpha}{\sqrt{\alpha}(2-\sqrt{\alpha})}$$

Notăm

$$J(\alpha) = \frac{\pi}{2} \int \frac{d\alpha}{\sqrt{\alpha}(2-\sqrt{\alpha})}$$

$$I(\alpha) = \frac{\pi}{2\sqrt{2}} \ln(2-\alpha) + J(\alpha)$$

Deci

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$$\sqrt{\alpha} = u \Rightarrow \frac{1}{2\sqrt{\alpha}} d\alpha = du$$

Deci

$$I(\alpha) = \frac{-\pi}{2\sqrt{2}} \int \frac{d\alpha}{2-\alpha} + \frac{\pi}{2} \int \frac{d\alpha}{\sqrt{\alpha}(2-\sqrt{\alpha})}$$

Notăm

$$J(\alpha) = \frac{\pi}{2} \int \frac{d\alpha}{\sqrt{\alpha}(2-\sqrt{\alpha})}$$

$$I(\alpha) = \frac{\pi}{2\sqrt{2}} \ln(2-\alpha) + J(\alpha)$$

$$\sqrt{\alpha} = u \Rightarrow \frac{1}{2\sqrt{\alpha}} d\alpha = du$$

$$J(u) = \int \frac{\pi}{2-u^2} du = \pi \int \frac{du}{2-u^2}$$

$$J(u) = \frac{\pi}{2\sqrt{2}} \int \left(\frac{1}{\sqrt{2}-u} + \frac{1}{\sqrt{2}+u} \right) du$$

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Deci

$$J(u) = \frac{\pi}{2\sqrt{2}} \left(-\ln(\sqrt{2}+u) + \ln(\sqrt{2}-u) + C \right)$$

$$J(u) = \frac{\pi}{2\sqrt{2}} \int \left(\frac{1}{\sqrt{2}-u} + \frac{1}{\sqrt{2}+u} \right) du$$

Deci

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Revenim la

$$J(\alpha) = \frac{\pi}{2\sqrt{2}} \left(-\ln(\sqrt{2}+\sqrt{\alpha}) + \ln(\sqrt{2}+\sqrt{\alpha}) + C \right)$$

$$I(\alpha) = \frac{\pi}{2\sqrt{2}} \ln \frac{(2-\alpha)(\sqrt{2}+\sqrt{\alpha})}{\sqrt{2}-\sqrt{\alpha}} + C$$

$$I(0) = \frac{\pi}{2\sqrt{2}} \ln 2 + C$$

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$$I(0) = 0 \Rightarrow C = \frac{-\pi}{2\sqrt{2}} \ln 2$$

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Aşadar

$$I(\alpha) = \frac{\pi}{2\sqrt{2}} \ln \frac{(2-\alpha)(\sqrt{2}+\sqrt{\alpha})}{\sqrt{2}-\sqrt{\alpha}} + \frac{-\pi}{2\sqrt{2}} \ln 2$$

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Aşadar

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$$I(\alpha) = \frac{\pi}{2\sqrt{2}} \ln \frac{(2-\alpha)(\sqrt{2}+\sqrt{\alpha})}{2(\sqrt{2}-\sqrt{\alpha})}$$

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Aşadar

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$$I(\alpha) = \frac{\pi}{2\sqrt{2}} \ln \frac{(2-\alpha)(\sqrt{2}+\sqrt{\alpha})}{2(\sqrt{2}-\sqrt{\alpha})}$$

$$I(1) = \frac{\pi}{2\sqrt{2}} \ln \frac{\sqrt{2}+1}{2(\sqrt{2}-1)}$$

$$\int_0^{\infty} \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx = ?$$

$$\int_0^{\infty} \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx = ?$$

$$\alpha = \int_0^{\infty} \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx = \int_0^{1} \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx + \int_1^{\infty} \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx$$

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Notăm

$$I = \int_0^{1} \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx$$

$$J = \int_1^{\infty} \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx$$

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$$\alpha = \int_0^\infty \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx = \int_0^1 \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx + \int_1^\infty \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx$$

Notăm

$$I = \int_0^1 \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx$$

$$J = \int_1^\infty \frac{\sin(2\ln^2 x)}{(1+x)\ln^2 x}dx$$

Deci $\alpha = I + J$

Dacă $x \mapsto \frac{1}{x} \Rightarrow dx \mapsto \frac{-1}{x^2}dx$

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$$J = \int_1^0 \frac{\sin(2\ln^2 \frac{1}{x})}{\left(1+\frac{1}{x}\right)\ln^2 x} \frac{-1}{x^2}dx$$

Dacă $x \mapsto \frac{1}{x} \Rightarrow dx \mapsto \frac{-1}{x^2} dx$

$$J = \int_1^0 \frac{\sin(2\ln^2 \frac{1}{x})}{\left(1 + \frac{1}{x}\right)\ln^2 x} \frac{-1}{x^2} dx$$

Deci

$$J = \int_0^1 \frac{\sin(2\ln^2 x)}{(x+1)x\ln^2 x} dx$$

Atunci

$$\alpha = \int_0^1 \frac{\sin(2\ln^2 x)}{(x+1)\ln^2 x} dx + \int_0^1 \frac{\sin(2\ln^2 x)}{(x+1)x\ln^2 x} dx$$

Atunci

$$\alpha = \int_0^1 \frac{\sin(2\ln^2 x)}{(x+1)\ln^2 x}dx + \int_0^1 \frac{\sin(2\ln^2 x)}{(x+1)x\ln^2 x}dx$$

$$\alpha = \int_0^1 \frac{\sin(2\ln^2 x)}{x\ln^2 x}dx$$

Atunci

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$$\alpha = \int_0^1 \frac{\sin(2\ln^2 x)}{x\ln^2 x}dx$$

Notăm $\ln x = t \Rightarrow \frac{1}{x}dx = dt$

Atunci

$$\alpha = \int_0^1 \frac{\sin(2\ln^2 x)}{(x+1)\ln^2 x}dx + \int_0^1 \frac{\sin(2\ln^2 x)}{(x+1)x\ln^2 x}dx$$

$$\alpha = \int_0^1 \frac{\sin(2\ln^2 x)}{x\ln^2 x}dx$$

Notăm $\ln x = t \Rightarrow \frac{1}{x}dx = dt$

$$\alpha = \int_{-\infty}^0 \frac{\sin(2t^2)}{t^2}dt$$

Definim

$$f : \mathbb{R} \times [0, \infty] \to \mathbb{R}; f(a, t) = \frac{\sin(at^2)}{t^2}$$

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$$f : \mathbb{R} \times [0, \infty] \to \mathbb{R}; f(a,t) = \frac{\sin(at^2)}{t^2}$$

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$$I(0) = \int_0^{\infty} \frac{\sin 0}{t^2} = 0$$

Definim

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$$I(0)=\int_0^{\infty}\frac{\sin 0}{t^2}=0$$

$$I'(a)=\int_0^{\infty}\frac{\partial f}{\partial a}(t,a)dt=\int_0^{\infty}\cos(at^2)dt$$

Folosim $e^{i\theta} = \cos\theta + i\sin\theta$ si $\cos(at^2) = \mathrm{Re}(e^{iat^2})$.

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$$I'(a) = \int_0^{\infty} \mathrm{Re}(e^{iat^2})dt = \mathrm{Re}\int_0^{\infty} e^{iat^2}dt$$

Folosim $e^{i\theta} = \cos\theta + i\sin\theta$ si $\cos(at^2) = \mathrm{Re}(e^{iat^2})$.

$$I'(a) = \int_0^{\infty} \mathrm{Re}(e^{iat^2})dt = \mathrm{Re}\int_0^{\infty} e^{iat^2}dt$$

$$I'(a) = \mathrm{Re}\left(\frac{1}{2}\sqrt{\frac{\pi}{ia}}\right) = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,\mathrm{Re}\left(\frac{1}{\sqrt{i}}\right) = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,\mathrm{Re}(i^{\frac{-1}{2}})$$

$$I'(a) = \frac{1}{2}\sqrt{\frac{\pi}{a}}\frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{\frac{\pi}{2}}a^{\frac{-1}{2}}\Big| \int$$

$$I'(a) = \frac{1}{2}\sqrt{\frac{\pi}{a}}\frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{\frac{\pi}{2}}a^{\frac{-1}{2}} \Big| \int$$

$$I(a) = \sqrt{\frac{\pi}{2}}\sqrt{a} + C = \sqrt{\frac{a\pi}{2}} + C$$

$$I'(a) = \frac{1}{2}\sqrt{\frac{\pi}{a}}\frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{\frac{\pi}{2}}a^{\frac{-1}{2}}\Big| \int$$

$$I(a) = \sqrt{\frac{\pi}{2}}\sqrt{a} + C = \sqrt{\frac{a\pi}{2}} + C$$

Cum $I(0) = 0 \Rightarrow C = 0$

$$I'(a) = \frac{1}{2}\sqrt{\frac{\pi}{a}}\frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{\frac{\pi}{2}}a^{\frac{-1}{2}}\Big| \int$$

$$I(a) = \sqrt{\frac{\pi}{2}}\sqrt{a} + C = \sqrt{\frac{a\pi}{2}} + C$$

Cum $I(0) = 0 \Rightarrow C = 0$

$$I(a) = \sqrt{\frac{a\pi}{2}}$$

$$I'(a) = \frac{1}{2}\sqrt{\frac{\pi}{a}}\frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{\frac{\pi}{2}}a^{\frac{-1}{2}} \Big| \int$$

$$I(a) = \sqrt{\frac{\pi}{2}}\sqrt{a} + C = \sqrt{\frac{a\pi}{2}} + C$$

Cum $I(0) = 0 \Rightarrow C = 0$

$$I(a) = \sqrt{\frac{a\pi}{2}}$$

$$I(2) = \sqrt{\pi}$$

Avantaje:
1)Simplificarea integralei prin introducerea unui parametru auxiliar
2)Rezolvarea integralelor dependente de parametri
3)Rezolvarea problemelor fără soluţii evidente prin integrare directă
Dezavantaje:
1)Complexitatea formalismului matematic
2)Dificultate în identificarea parametrului potrivit
3)Limitări în aplicabilitate

MULTUMESC PENTRU ATENŢIE!