Workshop for Young Researchers in Mathematics
13th edition

Iaşi, Romania, May 23-24, 2024

Sponsored by

EUROPEAN MATHEMATICAL SOCIETY

Organized by

Simion Stoilow Institute of Mathematics of the Romanian Academy

Octav Mayer Institute of Mathematics of the Romanian Academy, Iaşi Branch

Faculty of Mathematics of the Alexandru Ioan Cuza University

Faculty of Mathematics and Computer Science, Ovidius University of Constanţa

Analysis, PDEs & Applied Mathematics
– List of abstracts –

Low regularity well-posedness for the generalized surface quasi-geostrophic front equation

Ovidiu-Neculai Avădanei

University of California, Berkeley

Abstract: We consider the well-posedness of the generalized surface quasi-geostrophic (gSQG) front equation. By making use of the null structure of the equation, we carry out a paradifferential normal form analysis in order to obtain balanced energy estimates, which allows us to prove the local well-posedness of the g-SQG front equation in the non-periodic case at a low level of regularity (in the SQG case, this is only one half of a derivative above scaling). In addition, we establish global well-posedness for small and localized rough initial data, as well as modified scattering, by using the testing by wave packet approach of Ifrim-Tataru. This is joint work with Albert Ai.

Accelerated iterative algorithms for the Cauchy problem in steady-state anisotropic heat conduction

Mihai Bucătaru

University of Bucharest & ISMMA

Abstract: In the framework of stationary anisotropic heat conduction (the Laplace-Beltrami equation) without heat sources, we investigate both theoretically and numerically the acceleration of the two iterative algorithms of Kozlov et al. for the accurate, convergent and stable reconstruction of the missing temperature and normal heat flux on an inaccessible boundary of the domain occupied by a solid from the knowledge of Cauchy data on the remaining and accessible boundary. The inverse Cauchy problem is reformulated into an equivalent fixed-point problem, involving analysis of convergence criteria and determination of optimal relaxation parameters within a suitable function space. The numerical implementation is realised for two-dimensional homogeneous anisotropic solids using the finite element method and confirms a significant reduction in the number of iterations and hence CPU time required for the two relaxation algorithms proposed to achieve convergence, for both exact and perturbed Cauchy data. This is a joint work with Liviu Marin.

Pathological sums of Sobolev spaces and some counterexamples

Eduard Curcă

Alexandru Ioan Cuza University

Abstract: Suppose $Q \subset \mathbb{R}^d$ is a cube and W^{s_0,p_0}, $W^{\sigma,q}$, W^{s_1,p_1} are three Sobolev-Slobodeckii spaces such that $s_0, s_1 \geq 0$, $p_0, p_1 \in [1,\infty]$ and $\sigma := (1-\theta)\, s_0 + \theta s_1$, and $1/q := (1-\theta)/p_0 + \theta/p_1$ for some $\theta \in (0,1)$. We give a necessary and sufficient condition for the embedding

$$W^{\sigma,q}(Q) \hookrightarrow W^{s_0,p_0}(Q) + W^{s_1,p_1}(Q), \qquad (*)$$

to hold. By this we complement some results of P. Mironescu (2018). In the case $(*)$ does not hold, we indicate the construction of some counterexamples.

Isoperimetric problems on the square and hexagonal lattices

Andreea Dima

IMAR

Abstract: It is well-known that in $\mathbb{R}^2$ the ball minimizes the perimeter under area constraints. One may ask what happens when we replace $\mathbb{R}^2$ with the square and then with the hexagonal lattices. We prove that, given n squares of unit length forming a connected domain, the minimum perimeter is $2\lceil 2\sqrt{n}\,\rceil$ and that given n regular hexagons of unit length also forming a connected domain, the minimum perimeter is $2\left\lceil \sqrt{12n-3}\,\right\rceil$. Further open problems will be discussed. This is joint work with Liviu Ignat and it is partially based on the article *Maximal and minimal polyhexes* by Winston C. Yang.

Mixed Dirichlet-Robin problem for coupled anisotropic Darcy-Forchheimer-Brinkman equations

Andrei Gasparovici

Babes-Bolyai University

Abstract: We study mixed Dirichlet-Robin boundary value problems for the nonlinear anisotropic Darcy-Forchheimer-Brinkman system and a system of coupled Darcy-Forchheimer-Brinkman equations using a variational approach and fixed-point techniques. We also provide applications and numerical results related to viscous incompressible fluid flows in multidisperse porous media.

Approximations of the best constants

Liviu Ignat

IMAR & ICUB, University of Bucharest

Abstract: In this talk we consider some well known quotients related with either eigenvalue problems, Sobolev or Hardys inequality. We consider the infimum of these quotients and their discrete analogous in a finite element subspace. We estimate the difference between the best constants above as the discretization parameter goes to zero.

Jensen-Steffensen's inequalities in the context of majorization theory and in spaces with a curved geometry

Geanina Maria Lăchescu

University of Craiova

Abstract: The Jensen inequality for convex functions holds under the assumption that all of the included weights are nonnegative. If we allow some of the weights to be negative, then we get the Jensen-Steffensen inequality for convex functions. In this talk we extend the Jensen-Steffensen inequality on $\mathbb{R}^n$ via majorization ordering and derive similar inequalities for h_2-strongly convex functions (also Sherman's type inequalities).

Atmospherical frontogenesis in vortex liquid mixing and optimal placement

Dragoş Manea

IMAR

Abstract: In 1984, Charles Doswell introduced a mathematical description of the circular vortices that appear in the atmosphere, determined by variations in pressure and temperature. These circular motions generate a considerable amount of air mixing, hence increasing the homogeneity of the atmosphere in their neighborhood.

Starting from these considerations, the main question that our presentation addresses is how this mixing technique created by nature can be used in industrial applications, in order to mix together various flowing substances (liquids, sand, etc.)

Nevertheless, it is obvious that, in the sky, the air is mixed by a large number of vortices that have an interconnected effect that increases the circulation of gas. An interesting question then arises: are several connected vortices more effective in increasing the amount of mixing than one single bigger vortex? It will turn out that, in order to build an optimal

mixer, one should rather concentrate the whole mixing power in a single vortex placed on the interface between the fluids, instead of constructing several small stirrers.

Throughout the presentation, we will also address more advanced questions regarding the optimal placement of the mixers, looking for a fixed setup that acquires the best compromise between several initial configurations of the fluids that are to be stirred. The optimal setup we have achieved consists of two mixers that are placed in between the possible locations of the interface that separates the fluids.

A 2D + time mathematical model for multiphase optimization of air traffic trajectory and sequence

Radu-Adrian Mihai

Politehnica University of Bucharest

Abstract: We present an initial two spatial dimensions plus time mathematical model for the optimization of both the trajectory and arrival sequence of a set of aircraft with different initial states to a common final position. The problem is defined as a multiple phase optimal control problem, that minimizes the time required for the aircraft to reach the target.

Due to the global optimal solution being required at each step, the problem is sensitive to initial conditions. The requirement of fast computation also excludes the use of global optimization methods. Thus, we intend to find a method that avoids local solutions by penalizing the fast variation of the controls and by providing a starting point that is close to the global optimal solution.

On a Mandelbrot-like set for a family of exponential maps

Dan Paraschiv

IMAR & IMFM Ljubljana

Abstract: We consider the family $F_{\lambda,m}(z) = \lambda z^m \exp(z)$, for $m \geq 2$. There are precisely 2 critical points, $z = 0$ which is always a superattracting fixed point, and $z = -m$ which is a free critical point. The capture zones of the parameter plane, i.e. when the free critical point belongs to the basin of attraction of $z = 0$, have been previously studied by Fagella, Garijo, Jarque and Rocha. We use quasiconformal surgery to study the main Mandelbrot-like set, for which the free critical point does not belong to the basin of $z = 0$.

Self improving size estimates in compensated compactness

Bogdan Raiţă

Georgetown University

Abstract: We review some recent results in compensated compactness, concerning primarily concentration effects of pde constrained sequences. We show that Müller's $L \log L$ bound

$$\Phi(Du) \geq 0,\ Du \in L^q(\mathbb{R}^n) \implies \Phi(Du) \in L \log L_{loc}$$

for $\Phi = \det$ and $q = n$ holds for quasiconcave Φ which are homogeneous of degree $q > 1$. This contrasts similar Hardy bounds which hold only for null Lagrangians.

Hardy-Rellich Inequalities in domains of $\mathbb{R}^N$ and applications

Teodor Rugină

University of Bucharest

Abstract: In this talk I will give a brief introduction regarding the Hardy inequalities in different frameworks in $\mathbb{R}^N$ and connections to the Rellich inequality. The main focus will be on inequalities posed in a domain and with singular potential expressed as powers of distance to the boundary functions. I will also present recent results in this topic and applications to spectral theory.

The limiting behavior of constrained minimizers in variable exponent spaces

Anisia Teca

University of Craiova

Abstract: In this talk we discuss the asymptotic behavior of the sequences of nonnegative minimizers for two families of constrained minimization problems considered on a given open and bounded set from the Euclidean space $\mathbb{R}^N$ ($N \geq 1$) and involving variable exponent growth conditions. In both cases we show that the limit is the distance function to the boundary of the domain normalized in a given Lebesgue-type space involved in the construction of the two problems. In particular, our results complement to the case of variable exponent analysis similar studies available in the literature. This is a joint with Mihai Mihăilescu.