# Workshop for Young Researchers in Mathematics 13th edition

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Simion Stoilow Institute of Mathematics of the Romanian Academy (IMAR)

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# Algebra, Geometry, Number Theory & Topology - List of abstracts -

Gap results for biharmonic submanifolds in Euclidean spheres

### Ştefan Andronic

Alexandru Ioan Cuza University

Abstract: We will begin by introducing the notion of biharmonic submanifolds and presenting some gap results of the mean curvature for biharmonic submanifolds with parallel mean curvature vector field (PMC) in Euclidean spheres. Next, we will determine a larger gap of the mean curvature for a class of PMC proper biharmonic submanifolds in spheres. When the bounds of the gap are reached, we obtain splitting results of the submanifold.

On the spectrum of the Dirac operator on degenerating Riemannian surfaces and the Selberg Zeta function

## Cipriana Anghel

Mathematisches Institut Göttingen & IMAR

Abstract: We study the behavior of the spectrum of the Dirac operator on degenerating families of Riemannian surfaces, when the length of a simple closed geodesic shrinks to zero. We work under the hypothesis that the spin structure along the pinched geodesic is non-trivial. It is well-known that the spectrum of an elliptic differential operator on a compact manifold varies continuously under smooth perturbations of the metric. The difficulty of our problem arises from the non-compactness of the limit surface, which is of finite area with two cusps. The main tool for this investigation is to construct an adapted pseudodifferential calculus (in the spirit of the celebrated *b*-algebra of Melrose) which includes both the family of Dirac operators on the family of compact surfaces and the Dirac operator on the limit (non-compact) surface, together with their resolvents. Milnor formula for smoothable curve singularities

## Andrei Benguş-Lesnier

Institute of Mathematics and Informatics of the Bulgarian Academy of Science

Abstract: My talk will revolve around recent work joint with Antoni Rangachev about germs of complex reduced curve singularities that are smoothable. They are those who can be seen as a special fiber of a flat family of curves over a one parameter base space, for which the generic fibre is smooth. Our main achievement is a formula for the Milnor number of such a germ, generalising a formula of Lê, Teissier and Greuel. What is remarkable about this result is the the expression of a topological invariant of a singularity in terms of multiplicities of ideals, so purely algebraic expressions. We will present the details of each term of the formula as well as sketch out the approach to achieving our result.

Special non-Kähler metrics on Endo-Pajitnov manifolds

# Cristian Ciulică

University of Bucharest

Abstract: We investigate the metric and cohomological properties of higher dimensional analogues of Inoue surfaces, that were introduced by Endo and Pajitnov. We provide a solvmanifold structure and show that in the diagonalizable case, they are formal and have invariant de Rham cohomology. Moreover, we obtain an arithmetic and cohomological characterization of pluriclosed and astheno-Kähler metrics and show they give new examples in all complex dimensions.

On the Chermak-Delgado lattice of a finite group

### Georgiana Fasola

Alexandru Ioan Cuza University

Abstract: In this talk, we present some recent results on the Chermak-Delgado lattice associated to a finite group. More precisely, we classify the finite groups with a small number of subgroups not in the Chermak-Delgano lattice. We also study the subgroups of minimum Chermak-Delgado measure. Far-flung Gorenstein numerical semigroup rings

### Teodor-Ioan Grigorescu

University of Bucharest

Abstract: The far-flung Gorenstein property for one-dimensional Cohen-Macaulay local rings was introduced by D. Stamate, J. Herzog and S. Kumashiro (J. Algebra, 2023). They classified the far-flung Gorenstein numerical semigroup rings of type 2 and type 3. I am working to classify the far-flung Gorenstein numerical semigroup rings of type 4.

Brill-Noether Theory of vector bundles on surfaces

### Irene Macías Tarrío

University of Barcelona

Abstract: Let X be a smooth projective variety of dimension n over an algebraically closed field K of characteristic 0, and let  $M_H := M_{X,H}(r; c_1, \ldots, c_s)$  be the moduli space of rank-r vector bundles on X with respect to an ample divisor H on X and with fixed Chern classes  $c_i(E) := c_i$  for  $i = 1, \ldots, s$  with  $s := \min\{r, n\}$ . One way to study the geometry of these moduli spaces is by examining their subvarieties. Among them one can try to study Brill-Noether loci  $W_H^k(r; c_1, \ldots, c_s)$ , whose points correspond to stable vector bundles having at least k independent sections. In my talk, after introducing some background material, I will introduce these subvarieties and present some questions related to them. Finally I will focus my attention on new results concerning the non-emptiness of the Brill-Noether locus in the case of rank-2 stable vector bundles on ruled surfaces.

# On the switching rook polynomial of frame polyominoes Francesco Navarra

Sabanci University

Abstract: Polyominoes are plane figures, made up of squares of the same size joined edge by edge. In 2012 A. A. Qureshi connected them to Commutative Algebra, assigning to every polyomino  $\mathcal{P}$  the ideal of the inner 2-minors of  $\mathcal{P}$  in a suitable polynomial ring  $S_{\mathcal{P}}$ . In this talk we study the Hilbert series of the coordinate ring  $K[\mathcal{P}] = S_{\mathcal{P}}/I_{\mathcal{P}}$  of frame polyominoes, showing that the *h*-polynomial of  $K[\mathcal{P}]$  is equal to the so-called switching rook polynomial of  $\mathcal{P}$ . We provide a demonstrative technique whose key ingredient is a new interpretation of the well-known result of McMullen and Walkup about the *h*-vector of a shellable simplicial complex, in terms of non-attacking rooks. This talk is based on a joint work with R. Jahangir.

# Do the dual Miller-Morita-Mumford classes vanish in the homology of the big mapping class group?

### Martin Palmer-Anghel

IMAR

Abstract: The Mumford conjecture – a consequence of the Madsen-Weiss theorem – describes the rational homology of the mapping class groups  $Mod(\Sigma(g, 1))$  in the limit as g goes to infinity, in terms of the dual Miller-Morita-Mumford (MMM) classes. Instead of taking the colimit of the mapping class groups, one may instead take the colimit of the surfaces  $\Sigma(g, 1)$  themselves, to obtain an infinite-type surface  $\Sigma(\infty)$ , and consider its mapping class group  $Mod(\Sigma(\infty))$ , called the "big mapping class group". The structure of its homology is very mysterious, and very large: it is uncountably generated in every positive degree. There is a natural homomorphism from the colimit of  $Mod(\Sigma(g, 1))$  to  $Mod(\Sigma(\infty))$ , and one may wonder what its effect is on homology; in particular whether the dual MMM classes vanish on  $Mod(\Sigma(\infty))$ .

This is a special case of a more general question for any infinite-type surface S: does its mapping class group Mod(S) admit non-zero homology classes supported on a compact subsurface of S? We will give a complete answer to this question when S has non-zero genus (including the case  $S = \Sigma(\infty)$ ) and a partial answer when S has genus zero. This represents joint work with Xiaolei Wu.

## Semistability conditions defined by ample classes

### Mihai Pavel

#### IMAR

Abstract: We present a class of semistability conditions defined by a system of ample classes for coherent sheaves over a smooth projective variety. Under some necessary boundedness assumptions, we show the existence of a well-behaved chamber structure for the variation of moduli spaces of sheaves with respect to the change of semistability. This is joint work with Damien Mgy and Matei Toma.

# Quadratic Pairs over Schemes and the Canonical Quadratic Pair on Clifford Algebras

### **Cameron Ruether**

IMAR

Abstract: Quadratic pairs on central simple algebras over a field were introduced in The Book of Involutions in order to manage the problems which arise with orthogonal involutions in characteristic 2. They are used throughout the book to give a characteristic agnostic treatment of semisimple groups of type D. However, a notable exception occurs in the final chapters on triality where the base field is assumed to be of characteristic different from 2. The authors make this assumption because, as they write, "we did not succeed in giving a rational definition of the quadratic pair on [the Clifford algebra]". This problem was recently solved by Dolphin and Quéguiner-Mathieu, who defined the canonical quadratic pair while working over fields of characteristic 2. The notion of quadratic pairs has been extended by Calmés and Fasel beyond working over fields to the setting over schemes.

We show that there exists a canonical quadratic pair on Clifford algebras in this setting as well, which extends the definition of Dolphin and Quéguiner-Mathieu. However, due to some quirks of working over schemes, the approach of Dolphin and Quéguiner-Mathieu requires some modification. We will review the basic definitions and properties of quadratic pairs, over fields and over schemes, and then describe the modified construction of the canonical pair on Clifford algebras. We will outline the key properties which earn it the name "canonical" and which allow for the theory of triality to proceed over schemes.

Geometry of the factors of Fermat numbers

Lorenzo Sauras-Altuzarra

IMAR & Vienna University of Technology

Abstract: We will discuss conditions for a given number in order to be a divisor of a Fermat number. In addition, we will comment further connections of this topic with the geometry of numbers and with the theory of generalized Pillai equations.

Combinatorial study of morsifications of real univariate singularities

# Miruna-Ştefana Sorea

Lucian Blaga University of Sibiu

Abstract: We study a broad class of morsifications of germs of univariate real analytic functions. We characterize the combinatorial types of the resulting Morse functions, via planar contact trees constructed from Newton-Puiseux roots of the polar curves of the morsifications. This is based on a joint work with Arnaud Bodin, Evelia Rosa Garca Barroso and Patrick Popescu-Pampu.

Special non-Kähler metrics on solv and nilmanifolds

## Miron Stanciu

University of Bucharest & IMAR

Abstract: I will go over a few results about the existence of special non-Kähler metrics (e.g. locally conformally Kähler, locally conformally balanced, pluriclosed) and the interplay between them on nilmanifolds and, more generally, solvmanifolds. A big portion of my talk will focus on a type of solvmanifold that generalizes Inoue surfaces, defined by Endo and Pajitnov, that can be shown to provide new examples of non-Kähler structures under certain algebraic conditions.

Brill-Noether loci on Enriques surfaces

Andrei Stoenică

University of Bucharest

Abstract: This talk is based on joint work in progress with Irene Macías Tarrío and Călin Spiridon. After a short survey of the basics of Brill-Noether theory, we will exhibit proper Brill-Noether loci inside the moduli spaces of stable rank 2 bundles with fixed Chern classes of a certain type on an Enriques surface which is covered by a Jacobian Kummer surface.