Workshop for Young Researchers in Mathematics
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Organized by

Simion Stoilow Institute of Mathematics of the Romanian Academy

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Octav Mayer Institute of Mathematics of the Romanian Academy, Iaşi Branch

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Faculty of Mathematics of the Alexandru Ioan Cuza University

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Faculty of Mathematics and Computer Science, Ovidius University of Constanţa

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On some Robin-transmission problems for the Brinkman system and a Navier-Stokes type system

Andrei Albisoru

Babes-Bolyai University

Abstract: The aim of the talk is to give well-posedness results for a Robin-transmission-type problem for the Brinkman system and a Robin-Dirichlet problem for the Brinkman system in bounded Lipschitz domains in $\mathbb{R}^n$, for $n \geq 3$. This is achieved by employing the methods of layer potential theory and Fredholm operator theory. We also establish well-posedness results for the Robin- transmission-type problem for the Brinkman system and a Robin-Dirichlet problem for the Darcy-Forchheimer-Brinkman system in bounded Lipschitz domains in $\mathbb{R}^3$. Joint work with M. Kohr, I. Papuc and W. L. Wendland.

Optimal control problem for a nonlinear Fokker-Planck equation via inputs with nonlocal action

Stefana Anita

Octav Mayer Institute of Mathematics of the Romanian Academy

Abstract: This talk concerns an optimal control problem (P) related to a nonlinear Fokker-Planck equation with inputs with nonlocal action. The problem is deeply related to a stochastic optimal control problem (P_S) for a McKean-Vlasov equation. The existence of an optimal control is obtained for the deterministic problem (P). The existence of an optimal control is established and necessary optimality conditions are derived for a penalized optimal control problem (P_h) related to a backward Euler approximation of the nonlinear Fokker-Planck equation (with a constant discretization step h). Passing to the limit one derives the necessary optimality conditions for problem (P).

Cauchy inverse problem with non-smooth boundary data in steady-state anisotropic heat conduction

Mihai Bucataru

Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy & University of Bucharest

Abstract: We investigate a situation that usually occurs for numerous real life problems in science and engineering, namely the reconstruction of the missing non-smooth thermal boundary conditions (i.e. temperature and normal heat flux) on an inaccessible portion of the boundary of an anisotropic solid from the knowledge of over-prescribed data (i.e. Cauchy data) on the remaining and accessible boundary in the steady-state case. It is well-known that the solution to this inverse Cauchy problem, in a very weak sense in this case, is unstable, provided that such a solution exists. Consequently, a stabilizing method is developed herein based on a priori knowledge on the solution to this Cauchy problem and the smoothing nature of the corresponding direct problems employed. More precisely, this inverse problem is approached by transforming it into a control one which reduces to solving an appropriate minimization problem in a suitable function space. The latter is approached by employing an appropriate variational method which yields a gradient-based iterative algorithm that consists, at each step, of two direct and two corresponding adjoint problems. It is worth mentioning that this algorithm is designed to approximate merely square integrable boundary data, hence the notion of solution and the convergence analysis require special attention. The numerical implementation is realized for various two-dimensional non-homogeneous anisotropic solids using the finite element method (FEM), whilst regularization is achieved by ceasing the iterative process according to three stopping regularizing criteria.

Stability results for uncertainty principles

Cristian Cazacu

University of Bucharest & Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy

Abstract: We provide optimal constants for the stability inequalities for well-known uncertainty principles and a class of Caffarelli-Kohn-Nirenberg inequalities. For the Heisenberg uncertainty principle we introduce a new deficit function different from what is considered in the literature and then obtain the best constants. In addition, we recover the stability inequalities with optimal constants with respect to the deficit functions considered in the literature. This talk is based on several joint works. Partially supported by the grant no. PN-III-P1-1.1-TE-2021-1539, CNCS - UEFISCDI, Romania.

Some Bourgain-Brézis type solutions via complex interpolation

Eduard Curca

Alexandru Ioan Cuza University

Abstract: In 2002 Bourgain and Brézis proved that given a vector field

$$v \in \mathcal{S}'(\mathbb{R}^d) \cap \dot{W}^{1,d}(\mathbb{R}^d)$$

there exists a vector field $u \in L^\infty(\mathbb{R}^d) \cap \dot{W}^{1,d}(\mathbb{R}^d)$ such that $\operatorname{div} u = \operatorname{div} v$. We prove several results of a similar nature in which we take into consideration the Fourier support of the solutions. For instance, in the case $d \geq 3$ we prove the following: for any vector field $v \in \mathcal{S}'(\mathbb{R}^d) \cap \dot{B}_q^{d/p,p}(\mathbb{R}^d)$ (where $p \in [2, \infty)$ and $q \in (1, 2)$), with *supp* $\widehat{v} \subseteq \mathbb{R}^d \backslash (-\infty, 0)^d$, there exists a vector field $u \in L^\infty(\mathbb{R}^d) \cap \dot{B}_2^{d/p,p}(\mathbb{R}^d)$, with *supp* $\widehat{v} \subseteq \mathbb{R}^d \backslash (-\infty, 0)^d$, such that

$$\operatorname{div} u = \operatorname{div} v,$$

and

$$\|u\|_{L^\infty \cap \dot{B}_2^{d/p,p}} \lesssim \|v\|_{\dot{B}_q^{d/p,p}}.$$

Our arguments rely on a version of the complex interpolation method combined with some ideas of Bourgain and Brézis.

Solutions for nonlinear elliptic equations with singular potentials

Maria Farcaseanu

Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy

Abstract: In this talk, we present recent results on the existence of solutions for some nonlinear elliptic equations with singular potentials. This is joint work with Florica Cîrstea. The presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P1-1.1-PD-2021-0037.

Dirichlet problems for the anisotropic Darcy-Forchheimer-Brinkman system and related models

Andrei Gasparovici

Babes-Bolyai University

Abstract: We study Dirichlet boundary value problems for the nonlinear anisotropic Darcy-Forchheimer-Brinkman system and a system of two coupled Darcy-Forchheimer-Brinkman equations using variational methods and fixed point techniques. We also provide applications related to viscous incompressible fluid flows in monodisperse and bidisperse porous media.

On the flag and S-curvatures on the interpolated Poincaré metric

Sándor Kajántó

Babes-Bolyai University

Abstract: A disk $\mathbb{D}$ is endowed with a Poincaré-type Randers metric $F_\lambda, \lambda \in [0,1]$, that "linearly" interpolates between the usual Riemannian Poincaré disc model ($\lambda = 0$, constant sectional curvature -1 and zero S-curvature) and the Finsler-Poincaré metric ($\lambda = 1$, constant flag curvature $-1/4$ and constant S-curvature with isotropic factor $1/2$), respectively. We show that, for some particular choices of the flagpoles, when $\lambda \nearrow 1$, both the flag and normalized S-curvatures of the metric F_λ blow up close to the boundary of the disk.

Time-periodic weak solutions for an incompressible Newtonian fluid interacting with an elastic plate

Claudiu Mindrila

Charles University, Prague

Abstract: Consider a smooth bounded 3D domain containing a Newtonian fluid which verifies the Navier-Stokes equations. To a flat part of this reference domain, an elastic membrane is attached and is allowed to move only in normal (vertical) direction. We assume that, on the boundary, the velocity of the fluid equals the velocity of the membrane. The system evolves under the action of external time-periodic forces. Provided that the magnitude of the forces and the volume of the domain remain sufficiently small, we prove that at least one time-periodic weak solution exists. This is a joint result with S. Schwarzacher (Uppsala Universitet & Charles University).

Achievable connectivities of Fatou components for a family of rational maps

Dan Paraschiv

Universitat de Barcelona

Algebraic differential operators

Iulia Plesca

Alexandru Ioan Cuza University

Abstract: We look at conditions for some second order differential operators to not have logarithmic singularities.

Gaussian Processes for Systems of Linear PDEs with Constant Coefficients

Bogdan Raita

Scuola Normale Superiore di Pisa

Abstract: Including PDEs into machine learning models is an important way of incorporating physical knowledge. Given any system of linear PDEs with constant coefficients, we propose a family of Gaussian process (GP) priors such that all realizations are exact solutions of this system. We apply the Ehrenpreis-Palamodov fundamental principle, which works like a non-linear Fourier transform, to construct GP kernels. Our approach can infer probable solutions of linear PDE systems from any data such as noisy measurements, or initial and boundary conditions. We demonstrate our approach on three families of systems of PDEs, the heat equation, wave equation, and Maxwell's equations, where we improve upon the state of the art in computation time and precision, in some experiments by several orders of magnitude. Joint work with Marc Härkönen and Markus Lange-Hegermann.

H^{∞}-control problem for parabolic systems with singular inverse-square potential and convection

Teodor Rugina

University of Bucharest

Abstract: We present theH^{∞}-control problem with state feedback for infinite dimensional boundary control systems of parabolic type with distributed controls and disturbances. This method reduces the problem of finding an optimal control for the system to solving one particular algebraic Riccati equation. Then, we will see some applications of these results to equations with Hardy potentials and a convection term, in the case of a distributed control.

Perturbative Methods and Proper Elements for the Satellite and Space Debris Problem

Tudor Vartolomei

Alexandru Ioan Cuza University

Abstract: Proper elements are quasi-invariants of a Hamiltonian system, obtained through a normalization procedure. Proper elements have been successfully used to identify families of asteroids, sharing the same dynamical properties. For the space debris problem, the normalization procedure is an iterative method of reducing the initial Hamiltonian system to a simplified form, by removing the fast and semi-fast angles of the model which describes the dynamics of the system. It is worth mentioning that in the space debris problem, the perturbations to be included in the model depend on the altitude of the space debris. We show that the proper elements are particularly useful in the classification and back-tracing of space debris, as well as in the possible recognition of their origin. The results are supported by appropriate simulations and data analysis. Work in collaboration with Alessandra Celletti and Giuseppe Pucacco.