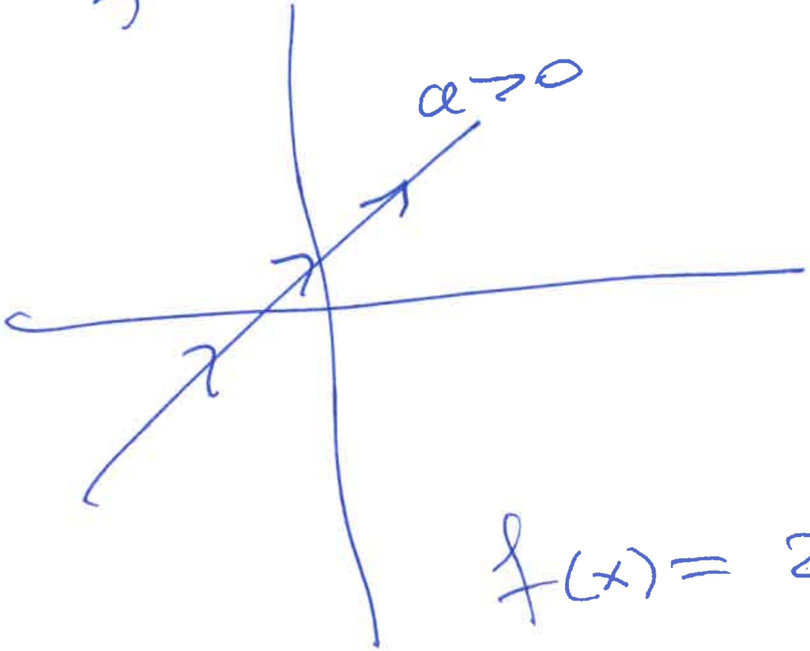
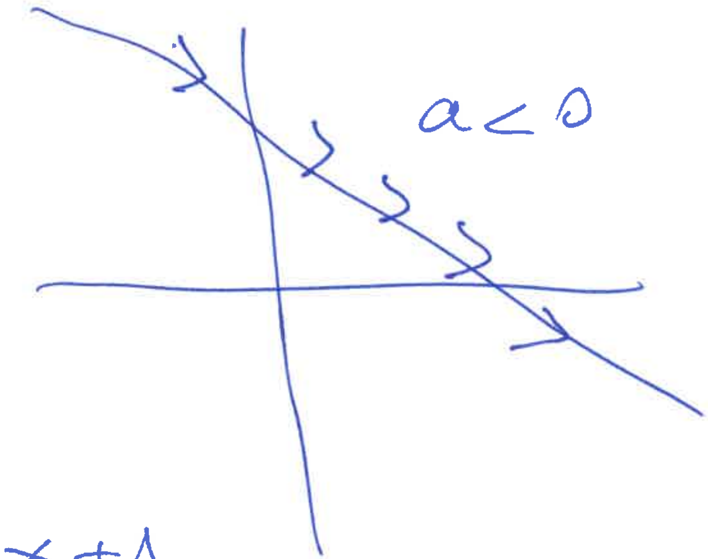
Funcția de gradul I

8.04.2023
ora 11^{00}

$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = ax + b, \; a \neq 0$





$f(x) = 2x + 1$

x	0	1
$2x+1$	1	3

Semn

x		$-b/a$	
$ax+b$	semn contrar lui a	0	semnul lui a

$2x-1<0$

$2x-1=0$

$2x=1$

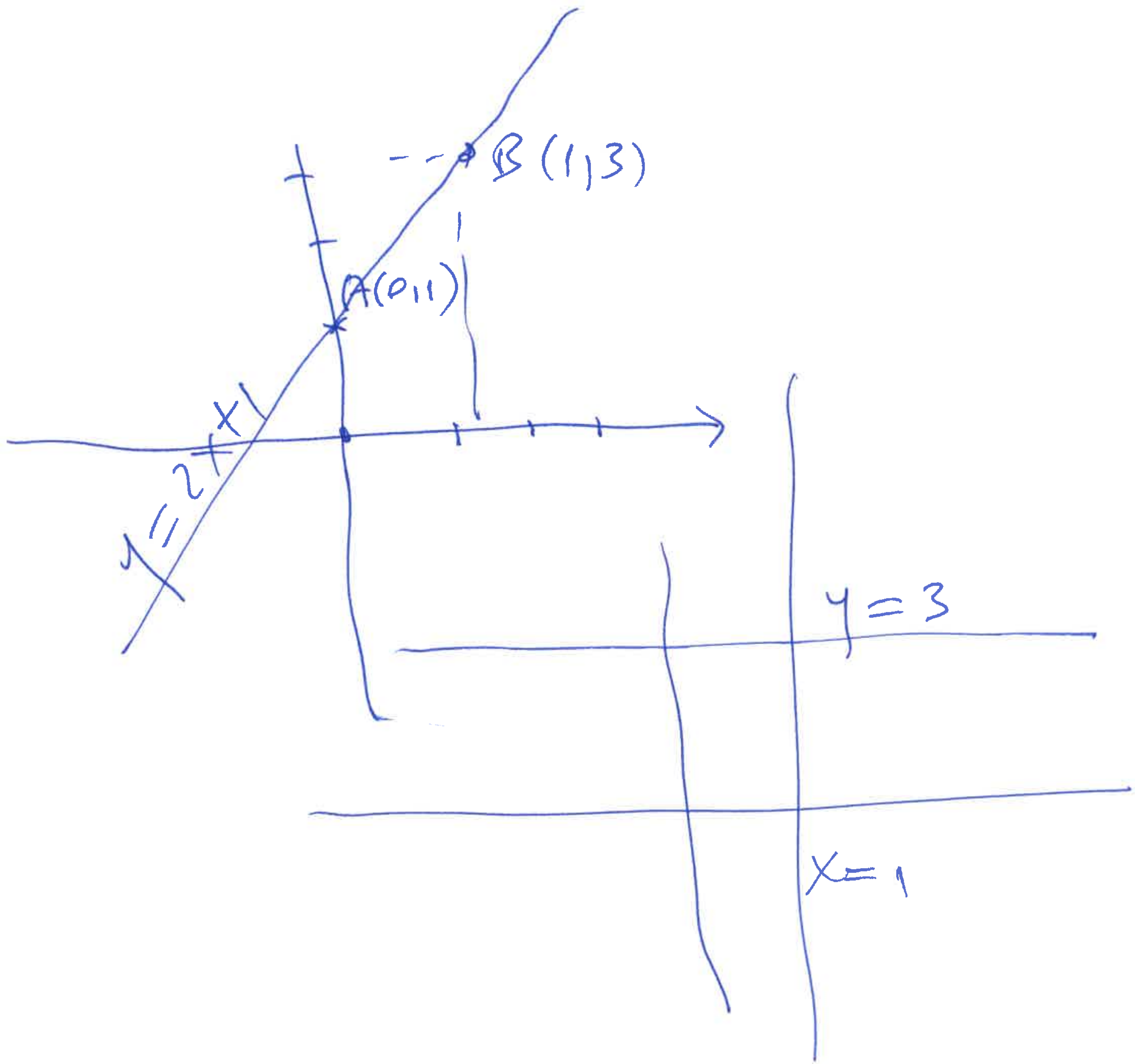
$x=\frac{1}{2}$

$a=2$

x		$\frac{1}{2}$	
$2x-1$	$- - - -$	0	$+ + + + + +$

x		0		$\frac{1}{2}$	
$2x-1$	$- -$	-1	$-$	0	$+ + + +$

$f(0)=2\cdot 0-1=-1$



B(1,3)
A(0,1)
y = 2x + 1
y = 3
x = 1

$$\frac{1}{x-1} \le \frac{1}{2} \quad x-1)$$

2)

$$\frac{1}{x-1} - \frac{1}{2} \le 0$$

$$\frac{2-(x-1)}{2(x-1)} \le 0$$

$$\frac{2-x+1}{2(x-1)} \le 0$$

$$\frac{3-x}{2(x-1)} \le 0$$

$3-x=0 \Rightarrow x=3$

$2(x-1)=0 \Rightarrow x-1=0 \Rightarrow x-1=0$

$x=1$

x		1		3	
$3-x$	$+$	$+$	$+$	0	$-$
$2(x-1)$	$-$	0	$+$	$+$	$+$
$\frac{3-x}{2(x-1)}$	$-$	|	$+$	0	$-$

$x \in (-\infty, 1) \cup [3, +\infty)$

$|x| = 3 \iff x = \pm 3$ $\boxed{(V)}$

$|x| = 3 \iff x = -3$ sau $x = +3$ $\boxed{(VIII)}$

$$|x| = \begin{cases} +x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

I $\quad x \geq 0$

$|x| = 3$

$+x = 3$

$x = 3 \geq 0 \Rightarrow x = 3$ soluție

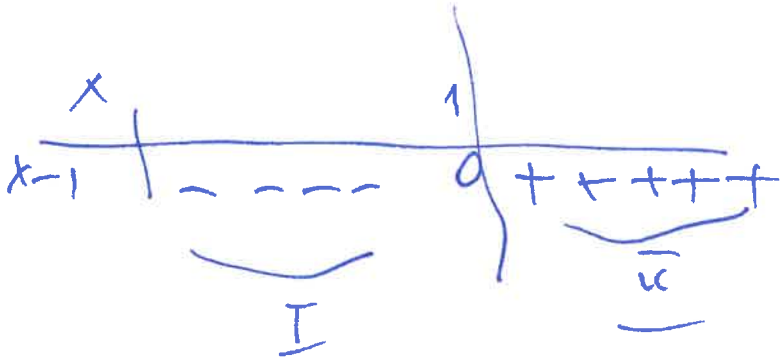
II $\quad x < 0$

$|x| = 3$

$-x = 3$

$x = -3 < 0 \Rightarrow x = -3$ soluție

$$|x-1| = 2$$



$$|x-1| = \begin{cases} +(x-1), & x \geq 1 \\ -(x-1), & x < 1 \end{cases}$$

I $\quad x < 1$

$$|x-1| = 2$$

$$-(x-1) = 2$$

$$1 - x = 2$$

$x = -1 < 1 \Rightarrow x = -1$ sol

II $\quad x \geq 1$

$$|x-1| = 2$$

$$+(x-1) = 2$$

$x = 3 \geq 1, \; x = 3$ soluție

$$|x-1|+|x-2|=4$$

x		1		2	
$x-1$	$-\ -$	0	$+\ +$	$+$	$+\ +$
$x-2$	$-\ -$	$-$	$-\ -$	0	$+\ +\ +$
	I		II		III

I $x \in (-\infty, 1)$

$$|x-1|+|x-2|=4$$
$$1-x+2-x=4$$
$$3-2x=4$$
$$2x=-1 \Rightarrow x=\frac{-1}{2} \in (-\infty, 1) \Rightarrow x=-\frac{1}{2} \text{ soluție}$$

II $x \in [1, 2)$

$$|x-1|+|x-2|=4$$
$$(x-1)+2-x=4$$
$$x-x+1=4$$
$$1=4 \Rightarrow x \in \emptyset$$

III $x \in [2, +\infty)$

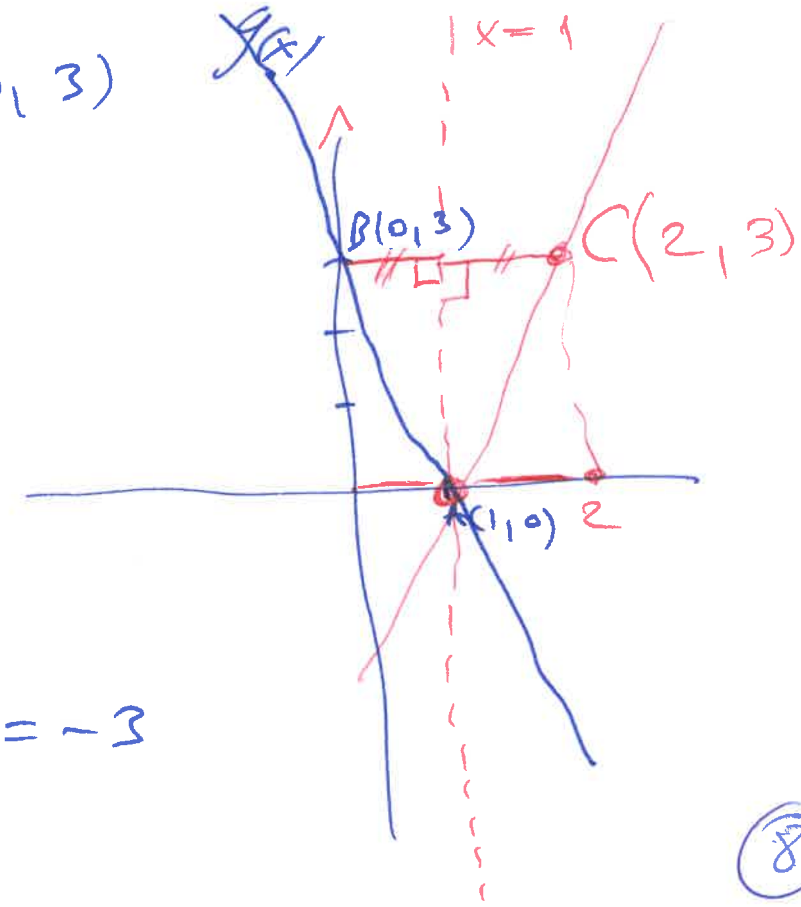
$$|x-1|+|x-2|=4$$
$$x-1+x-2=4$$
$$2x=7$$
$$x=\frac{7}{2} \in [2, +\infty)$$
$$\Rightarrow x=\frac{7}{2} \text{ soluție}$$

Determinați: $f:\mathbb{R}\to\mathbb{R}$ de grad I știind că graficul său și graficul funcției $g:\mathbb{R}\to\mathbb{R}$, $g(x)=-3x+3$ sunt simetrice față de dreapta $x=1$.

$g(x)=-3x+3$

$g(x)=0 \Rightarrow -3x+3=0 \Rightarrow x=1 \Rightarrow A(1,0)$

$x=0 \Rightarrow g(0)=-3\cdot 0+3=3 \Rightarrow B(0,3)$



$B(0,3)\in G_f \cap Oy \Rightarrow$

$\Rightarrow C(2,3)\in G_f$

$f(1)=0 \Rightarrow a+b=0$

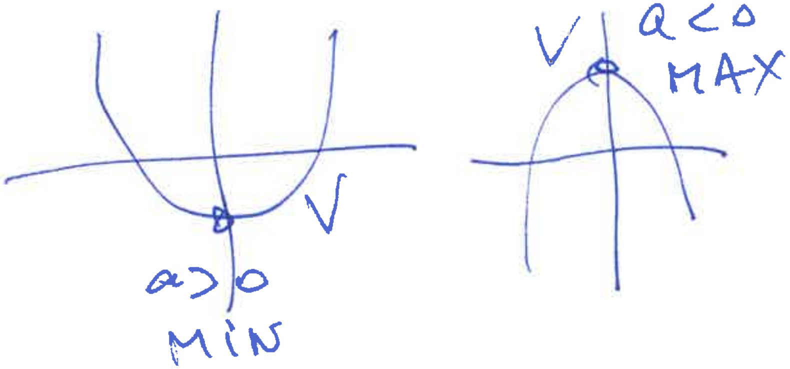
$f(2)=3 \Rightarrow 2a+b=3$ $\Rightarrow$

$\Rightarrow a=3,\ b=-3$

Fct gr II

$f:\mathbb{R}\to\mathbb{R}$

$f(x)=ax^2+bx+c,\ a\neq 0$



$\cap\, Ox \Rightarrow y=0$

$$ax^2+bx+c=0$$

$$\Delta\geq 0 \quad x_{1,2}=\frac{-b\pm\sqrt{\Delta}}{2a}$$

$$Im(f)=\{f(x)\mid x\in\mathbb{R}\}$$

imaginea fct.

Semn

$\Delta>0$

x		x_1		x_2	
ax^2+bx+c	$sgn(a)$	0	$-sgn(a)$	0	$sgn(a)$

$\Delta=0$

x		$x_1 = x_2$	
ax^2+bx+c	$sgn(a)$	0	$sgn(a)$

$\Delta<0$

x	
ax^2+bx+c	$sgn(a)$

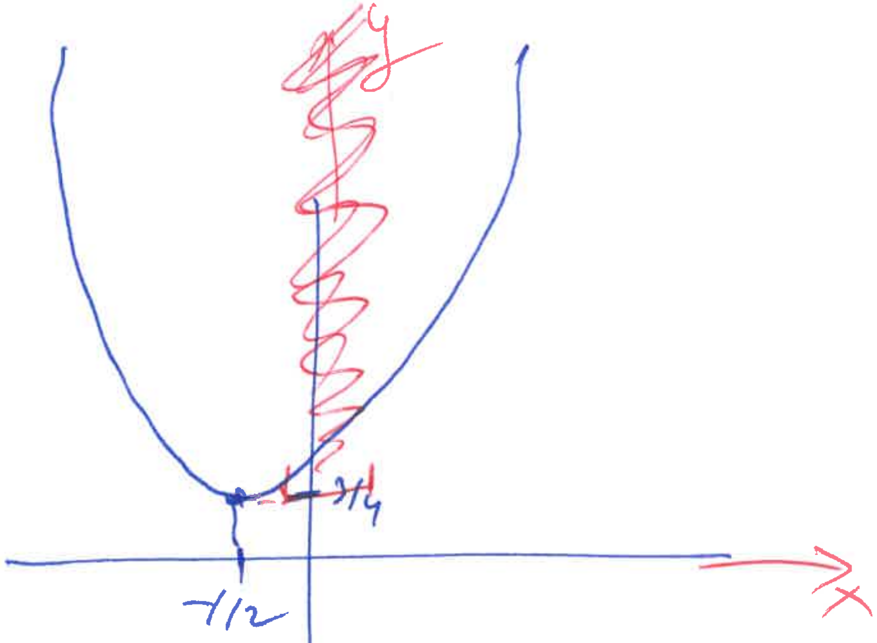
$$\Delta>0 \begin{cases} x_1+x_2=-\frac{b}{a} \\ x_1x_2=\frac{c}{a} \end{cases}$$

$f(x)=x^2+x+1$

$a=1>0 \Rightarrow$ MIN

$x_V = -\frac{b}{2a} = -\frac{1}{2}$

$y_V = -\frac{\Delta}{4a} = -\frac{(-3)}{4} = \frac{3}{4}$



Din desen $\Rightarrow \operatorname{Im} f = [3/4, +\infty)$

$f:\mathbb{R}\to\mathbb{R}$, $\quad f(x)=x^2+x+1$

? $Im(f)$

$Im(f)=\{f(x) \mid x\in\mathbb{R}\}$

$f(x)=y$

$x^2+x+1=y$

$x^2+x+1-y=0$ ec II în x trb. să aibe sol.

$a=1$

$b=1$

$c=1-y$

$\Downarrow$

$\Delta\geq 0$

$\Delta = 1-4(1-y) =$

$= 1-4+4y =$

$= 4y-3\geq 0 \Rightarrow y\geq \frac{3}{4}$

$y\in[3/4,+\infty)=Im(f)$

$\Delta > 0$

$$ax^2+bx+c = a(x-x_1)(x-x_2)$$

$$ax^2+bx+c = a(x^2 - x_1x - x_2x + x_1x_2)$$

$$\cancel{ax^2} + \underline{\underline{bx}} + c = \cancel{ax^2} \underline{\underline{- a(x_1+x_2)\cdot x}} + ax_1x_2$$

$$\begin{cases} b = -a(x_1+x_2) \\ c = a(x_1x_2) \end{cases} \Rightarrow \begin{cases} x_1+x_2 = -\dfrac{b}{a} \\ x_1x_2 = \dfrac{c}{a} \end{cases}$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - x - 2x + 2 = 0$$

$$x(x-1) - 2(x-1) = 0$$

$$(x-1)(x-2) = 0$$

$$x-1=0 \qquad x-2=0$$

$$x=1 \qquad x=2$$

$$ax^2+bx+c = a\underbrace{\left(x+\frac{b}{2a}\right)^2}_{=0} - \frac{\Delta}{4a}$$

$$x_V = -\frac{b}{2a} \quad | \quad y_V = \frac{\Delta}{4a}$$

$$(x+y)^2 = x^2+2xy+y^2$$

$$ax^2+bx+c=0$$

$$a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)=0$$

$$a\left(\underbrace{x^2+2\cdot\frac{b}{2a}\cdot x+\left(\frac{b}{2a}\right)^2}-\left(\frac{b}{2a}\right)^2+\frac{c}{a}\right)=0$$

$$a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+\frac{c}{a}\right]=0 \qquad 4a$$

$$a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}\right]=0 \qquad \Delta$$

$$\cancel{a}\left(x+\frac{b}{2a}\right)^2=\cancel{a}\cdot\frac{b^2-4ac}{4a^2} \qquad \Delta \geq 0$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{\Delta}}{2a}$$

$f, g : \mathbb{R} \to \mathbb{R}$

$$\begin{cases} f(x) + 2g(x) = 3x - 1 \\ 2f(x-1) - g(x-1) = x + 2 \end{cases} \qquad ?\ f(x), g(x)$$

$x \to x+1$ în a 2-a ecuație

$$2f(x+\cancel{1}-\cancel{1}) - g(x+\cancel{1}-\cancel{1}) = (x+1) + 2$$

$$\begin{cases} 2f(x) - g(x) = x + 3 \quad | \cdot 2 \\ f(x) + 2g(x) = 3x - 1 \end{cases}$$

$$4f(x) - 2g(x) = 2x + 6$$

$$5f(x) \quad / \quad = 5x + 5$$

$$f(x) = x + 1$$

? $m \in \mathbb{R}$ a.î.

$$x^2+y^2-4x-4y+m>0, \quad \forall\, x,y \in \mathbb{R}$$

$$\underbrace{x^2-4x+4}\ +\underbrace{y^2-4y+4}\ -8+m>0$$

$$\underbrace{(x-2)^2}_{\geq 0}+\underbrace{(y-2)^2}_{\geq 0}+m-8>0$$

$$\Downarrow$$

$$m-8>0$$

$$\underline{m>8}$$

Rezolv Δx

$$x^2-4x+y^2-4y+m>0$$

$a=1$

$b=-4$

$c=y^2-4y+m$

$$\Delta=(-4)^2-4(y^2-4y+m)<0$$

$$=-y^2+4y+4-m<0$$

$$\Delta y=32-4m<0 \Rightarrow \underline{m>8}$$

$$f(x) = \frac{x^2-4x+3}{x^2-2x+3} \qquad ?\ \mathrm{Im}(f)$$

Pt. ce valori ale lui m avem

$$(m-1)x^2 + mx + m+1 > 0 \quad \forall\, x \in \mathbb{R}.$$

$$\begin{cases} \Delta < 0 \text{ (acelasi semn)} \\ a > 0 \end{cases}$$

$\Delta = m^2 - 4(m-1)(m+1)$

$\Delta = m^2 - 4(m^2-1) =$

$= m^2 - 4m^2 + 4$

$= -3m^2 + 4 < 0$

$-3m^2 = -4$

$m^2 = \frac{4}{3}$

$m = \pm \frac{2}{\sqrt{3}}$

m	$-2/\sqrt{3}$		$2/\sqrt{3}$	
$-3m^2+4$	$- - - \ 0$	$+ + + +$	0	$- -$

$m \in \left(-\infty, -\frac{2}{\sqrt{3}}\right) \cup \left(2/\sqrt{3}, +\infty\right)$

$m > 1$

$m \in (1, +\infty)$

$$\frac{2}{\sqrt{3}} > 1 \Rightarrow 2 > \sqrt{3} \ \Big|^2$$

$$4 > 3.$$

$$\Rightarrow m \in \left(\frac{2}{\sqrt{3}}, +\infty\right)$$

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Rezolvați ecuația:

$$\min(x, x^2) + \max(x-1, x^2+1) = 3$$

$$\max(a,b) = \begin{cases} a, & a \geq b \\ b, & a < b \end{cases}$$

$$\max(x-1, x^2+1) = \begin{cases} x-1, & x-1 \geq x^2+1 \Rightarrow \ldots \\ x^2+1, & x-1 < x^2+1 \ldots \end{cases}$$

$$x^2+1 > x-1$$

$$x^2 - x + 2 > 0 \Rightarrow x \in \mathbb{R}.$$

$$\Delta = 1 - 8 < 0$$

$$\max(x-1, x^2+1) = x^2+1.$$