

Functia de gradul I. Functia de gradul al doilea.
Interpretarea geometrica a proprietatilor algebrice ale
functiei de gradul al doilea

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I. Functia de gradul I

Def: Functia $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(t) = at + b$,
cu $a, b \in \mathbb{R}$, $a \neq 0$, se numeste functie de gradul I.

Graficul functiei $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(t) = at + b$
este o dreapta cu puncta a.

Monotonie functiei de gradul I

- daca $a > 0$, atunci functia f este strict crescatoare;
- daca $a < 0$, atunci functia f este strict descrescatoare.

Semnul functiei de gradul I

x	$-\infty$	$-\frac{b}{a}$	$+\infty$
$f(x)$	$-\operatorname{sgn}(a)$	0	$\operatorname{sgn}(a)$

Exercitiul nr. 1: Determinati functia de gradul I care trece prin punctele A(1,2) si B(-1,0).

Solutie - Fie $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = ax + b$

$$\left. \begin{array}{l} A(1, 2) \in G_f \Leftrightarrow f(1) = 2 \Leftrightarrow a + b = 2 \\ B(-1, 0) \in G_f \Leftrightarrow f(-1) = 0 \Leftrightarrow -a + b = 0 \end{array} \right\}$$
$$\Rightarrow b = 1 \Rightarrow a = 1$$
$$f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x + 1 \quad \square$$

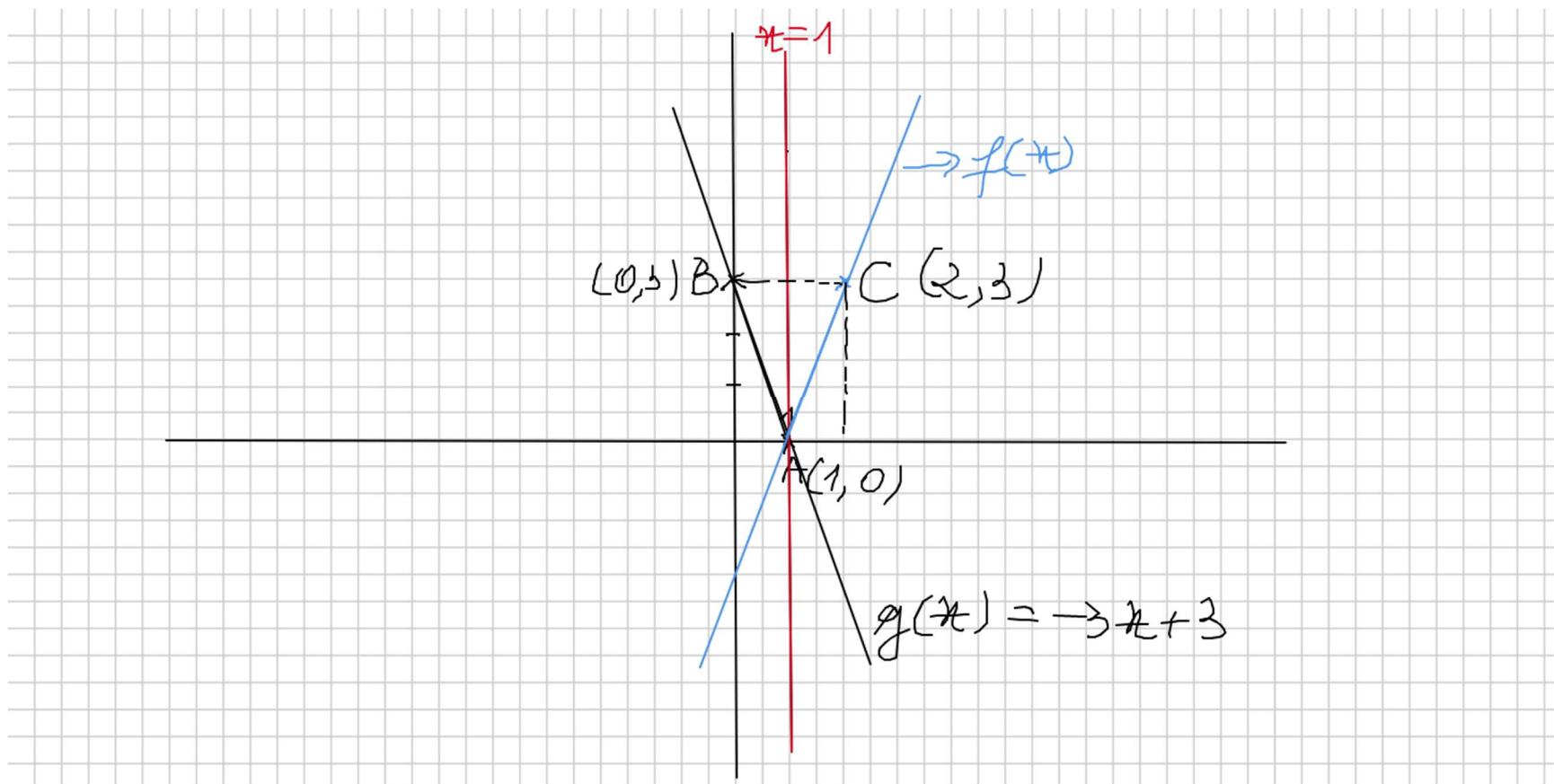
Exercitiul 2: Determinati functia $f: \mathbb{R} \rightarrow \mathbb{R}$, stiind ca graficul sau si graficul functiei $g: \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = -3x + 3$ sunt simetrice fata de dreapta $x=1$.
(Variante Bac 2009)

Solutie: $g: \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = -3x + 3$

$G_g \cap OX$: $g(x) = 0 \Rightarrow -3x + 3 = 0 \Rightarrow x = 1$

$G_g \cap OY$: $x = 0 \Rightarrow g(0) = 3 \Rightarrow B(0, 3)$

$A(1, 0)$



$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = ax + b$$

$$G_f \cap G_g \cap \{0\} = \{A(1, 0)\} \Rightarrow f(1) = 0$$

$$B(0, 3) \in G_g \cap \{0\} \Rightarrow C(2, 3) \in G_f \Leftrightarrow f(2) = 3$$

$$\begin{cases} a+b=0 \\ 2a+b=3 \end{cases} \Rightarrow a=3 \Rightarrow b=-3$$

Scì, $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3x - 3$. \square

II. Functia de gradul al doilea

Definitie: Functia $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = ax^2 + bx + c$, cu $a, b, c \in \mathbb{R}$, $a \neq 0$ se numeste functie de gradul al doilea.

Forma canonica a functiei de gradul al doilea:

$$f(x) = a x^2 + b x + c = a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{\Delta}{4a^2} \right],$$

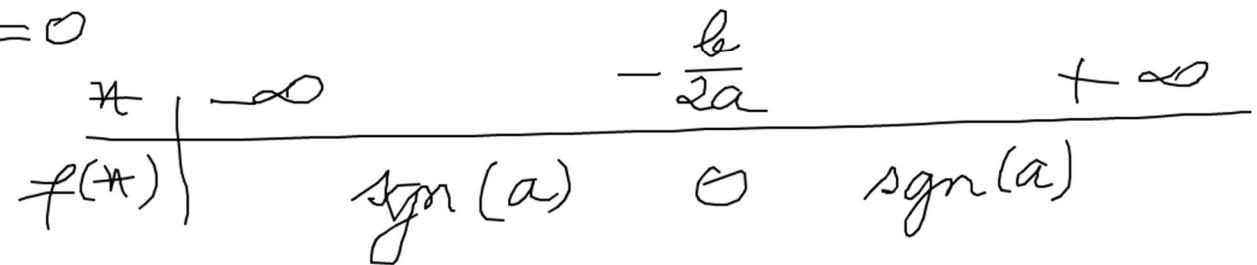
unde $\Delta = b^2 - 4ac$.

Semnul functiei de gradul al doilea:

Caz I: $\Delta < 0$



Caz II: $\Delta = 0$



Caz III: $\Delta > 0$

x	$-\infty$	x_1	x_2	$+\infty$
$f(x)$	$\text{sgn}(a)$	$0 - \text{sgn}(a) 0$		$\text{sgn}(a)$

Observații: ① $f(x) > 0, \forall x \in \mathbb{R} \Leftrightarrow \begin{cases} \Delta < 0 \\ a > 0 \end{cases}$

② $f(x) \geq 0, \forall x \in \mathbb{R} \Leftrightarrow \begin{cases} \Delta \leq 0 \\ a > 0 \end{cases}$

③ $f(x) < 0, \forall x \in \mathbb{R} \Leftrightarrow \begin{cases} \Delta < 0 \\ a < 0 \end{cases}$

Graficul functiei de gradul al doilea

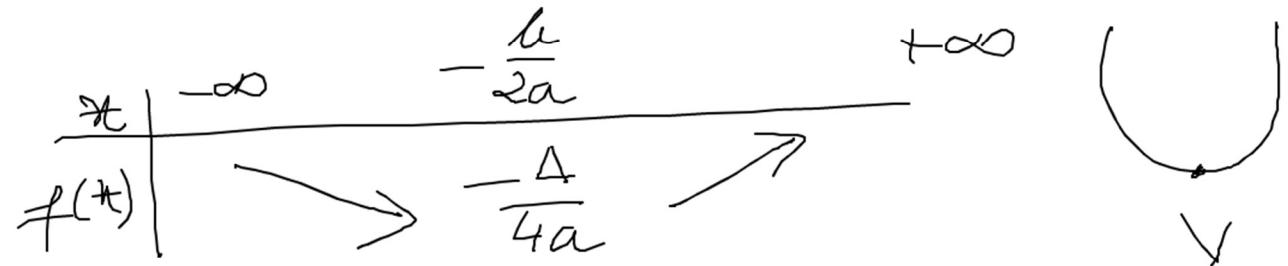
Graficul functiei $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = ax^2 + bx + c$ este o parabola de varf:

$$V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$$

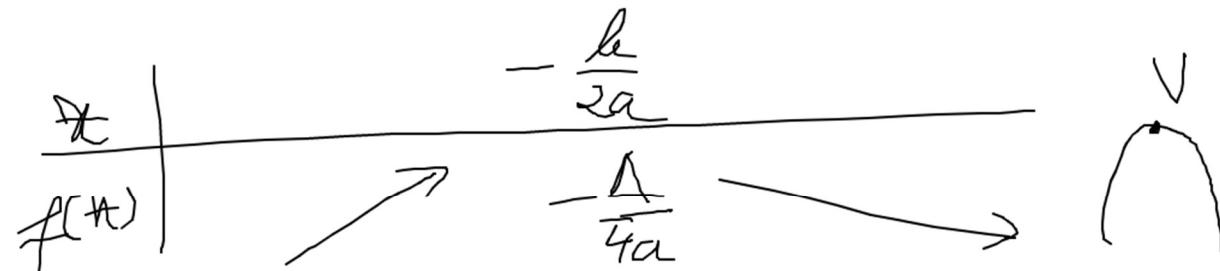

axa de simetrie $x = -b/(2a)$, care are ramurile orientate in sus, daca $a > 0$ si, respectiv, in jos, daca $a < 0$.

Monotonie si punctele de extrem ale functie de gradul al doilea

Caz 1: $a > 0$



Caz 2: $a < 0$



$$\boxed{\text{dacă } a > 0} \Rightarrow \min f = -\frac{\Delta}{4a}, x = -\frac{b}{2a} =$$

= punct de minim

- funcția este strict crescătoare pe $\left[-\frac{b}{2a}, +\infty\right)$,
- , și strict descreșătoare pe $\left(-\infty, -\frac{b}{2a}\right]$;
- ~~$\text{dacă } a < 0$~~ $\Rightarrow \max f = -\frac{\Delta}{4a}, x_{\max} = -\frac{b}{2a} =$
- = punct de maxim ;
- $x \in \left[-\frac{b}{2a}, +\infty\right) \Rightarrow f \downarrow_s; x \in \left(-\infty, -\frac{b}{2a}\right] \Rightarrow f \uparrow_s$

Relațiile lui Viète

$$at^2 + bt + c = 0, \quad t_1, t_2 = \text{racă.} \Rightarrow$$

$$\Rightarrow \begin{cases} t_1 + t_2 = -\frac{b}{a} \\ t_1 \cdot t_2 = \frac{c}{a} \end{cases}$$

Observatii:

$$\textcircled{1} \quad x_1^2 + x_2^2 = S^2 - P \quad , \quad S = x_1 + x_2$$

$$\textcircled{2} \quad x_1^3 + x_2^3 = S^3 - 3P \cdot S ;$$

- \textcircled{3} Ecuatia de gradul al doilea cu radacinile x_1 si x_2 este:
 $x^2 - Sx + P = 0$.

Exercitiu: Determinati multimea valorilor functiei $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2 + x + 1$.
(Bacalaureat 2011 - model subiect)

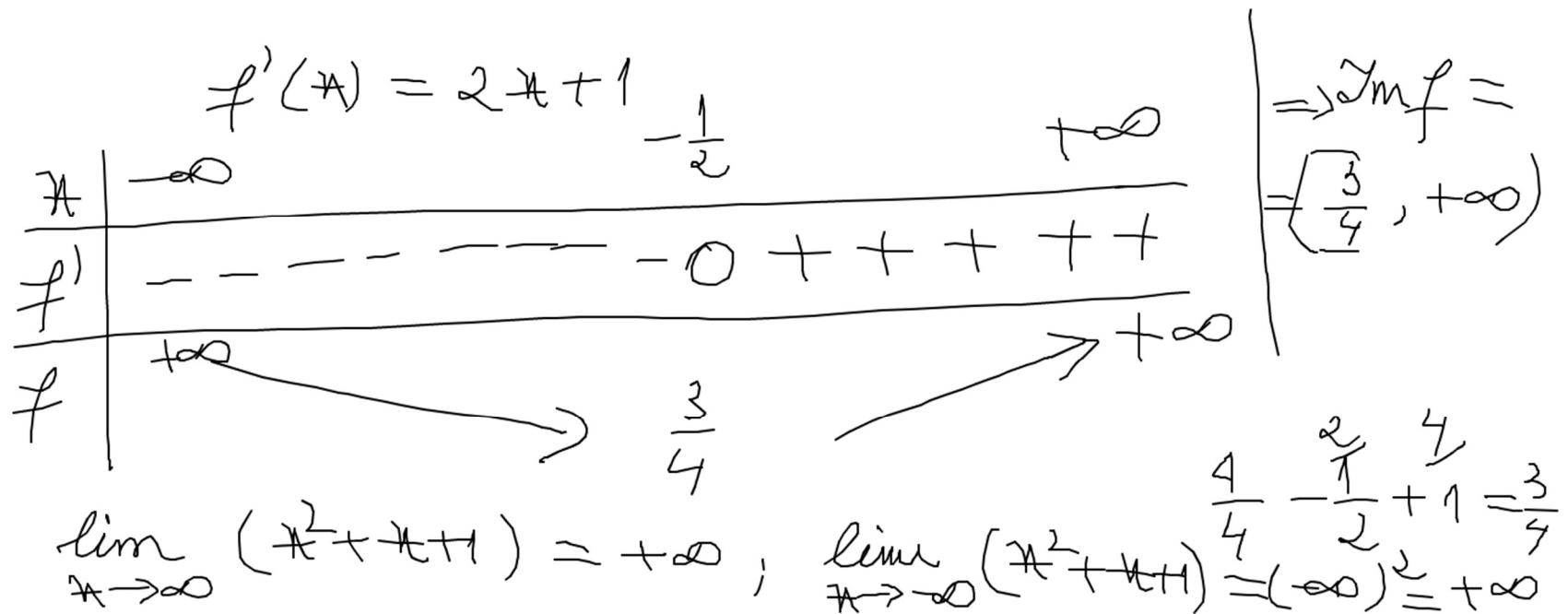
Solutie:
Metoda 1: $x^2 + x + 1 = 0$; $\Delta = -3 \Rightarrow x_1, x_2 \notin \mathbb{R}$

a=1, a > 0 $\Rightarrow V = \text{punct de minim}$

$$x_V = -\frac{b}{2a} = -\frac{1}{2} = \text{punct de minim} \quad \left| \begin{array}{l} \text{inf } f = \left[\frac{3}{4}, +\infty \right) \end{array} \right.$$

$$y_V = -\frac{\Delta}{4a} = \frac{3}{4} = \text{val min a fct}$$

Metoda I $f(x) = x^2 + x + 1$



Metoda III:

$$f(x) = y \Rightarrow x^2 + x + 1 - y = 0$$

Aceasta ecuație admete sol. reale $\Leftrightarrow \Delta \geq 0$

$$\Delta = 1 - 4 \cdot 1 \cdot (1 - y) = 1 - 4 + 4y = 4y - 3 \quad \left| \Rightarrow \right.$$

$$\Rightarrow 4y - 3 \geq 0 \Rightarrow y \geq \frac{3}{4} \Rightarrow f(x) \geq \frac{3}{4} \Rightarrow$$

$$\Rightarrow \text{Im } f = \left[\frac{3}{4}, +\infty \right)$$

Exercitiu: Fie ecuatie $x^2 + 2x + a = 0$ cu radacinile x_1 si x_2 .
 Determinati $a \in \mathbb{Z}$ stiind ca x_1, a, x_2 sunt in progresie aritmetica.

Solutie:

$$x_1, a, x_2 \text{ pr. aritm} \iff a = \frac{x_1 + x_2}{2} \iff$$

$$\iff 2a = x_1 + x_2$$

$$x_1^2 + 2x_1 + a = 0 \implies x_1 + x_2 = -\frac{2}{1} \left. \begin{array}{l} \implies 2a = -2 \\ \boxed{a = -1} \end{array} \right\} \quad \square$$

Exercitiu: Determinati m \in R pentru care ecuatia $x^2 - x + m^2 = 0$ are doua solutii reale egale.

(Bac, M1, 2010)

Solutie:

$$x_1 = x_2 \Leftrightarrow \Delta = 0 \Leftrightarrow 1 - 4m^2 = 0 \Leftrightarrow$$

$$x^2 - x + m^2 = 0$$

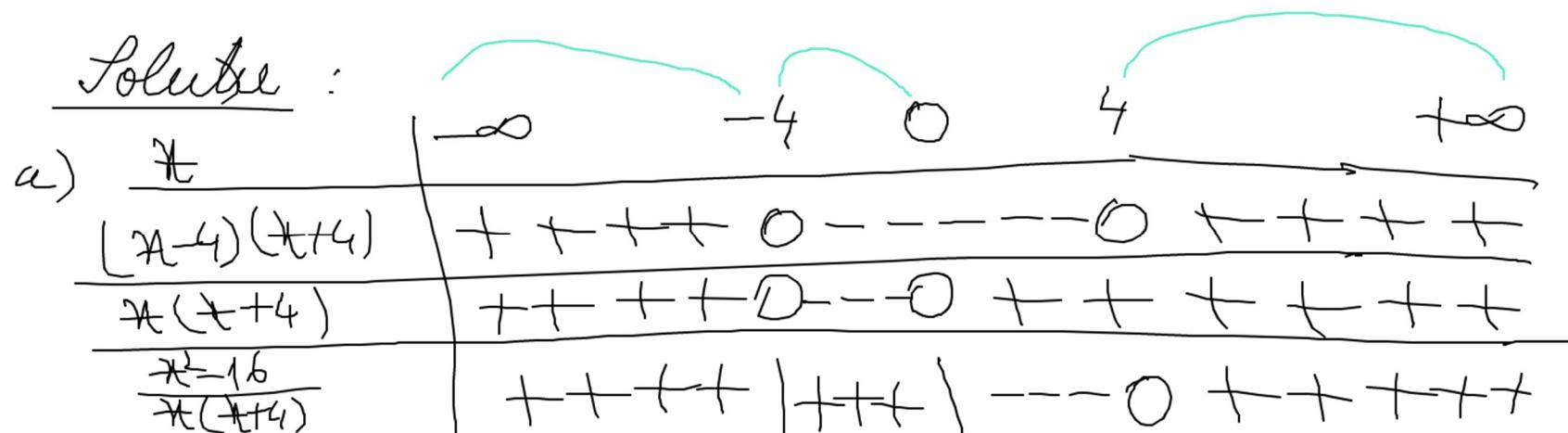
$$\Leftrightarrow 4m^2 - 1 \Leftrightarrow m^2 = \frac{1}{4} \Leftrightarrow m \in \left\{ \pm \frac{1}{2} \right\}$$

□

Exercitiu: Rezolvati in multimea numerelor reale inecuatiiile:

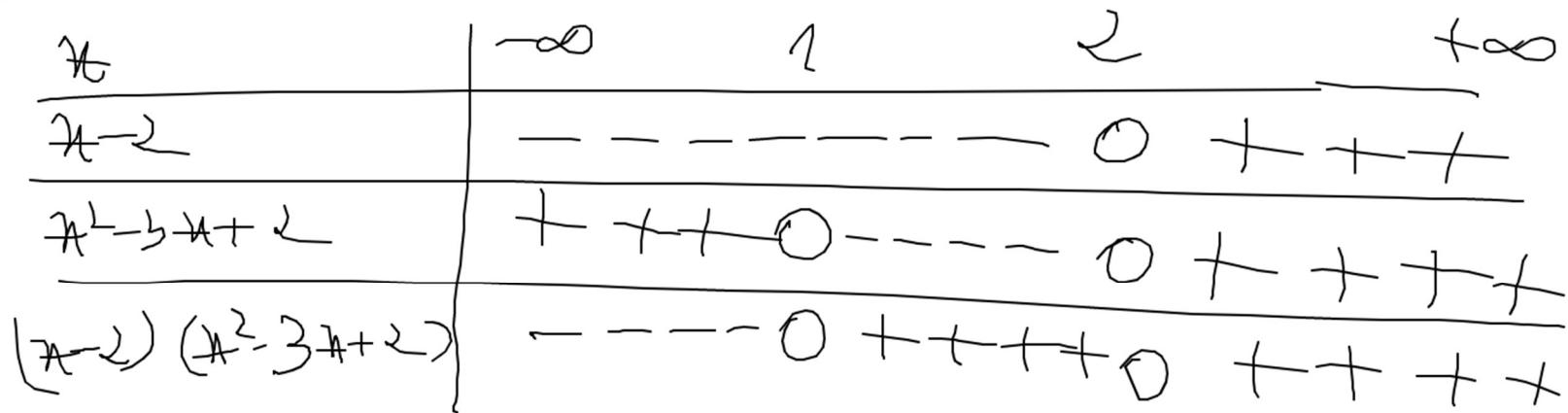
$$a) \frac{x^2 - 16}{x(x+4)} > 0 \quad ; \quad b) (x-2)(x^2 - 3x + 2) \leq 0.$$

Solutie :



$$\frac{x^2 - 16}{x(x+4)} > 0 \implies x \in (-\infty, -4) \cup (-4, 0) \cup (4, +\infty) \quad \square$$

6) $(x-2)(x^2-3x+2) \leq 0$



$$(x-2)(x^2 - 5x + 2) \leq 0 \implies x \in (-\infty, 1] \cup \{2\}$$



Exercitiu: Sa se determine functia de gradul I pentru care
 $f(f(x))=2f(x)+1$, oricare ar fi $x \in \mathbb{R}$.

Solutie:

- fie $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = ax + b$, $a \neq 0$
- $f(f(x)) = a(ax + b) + b = a^2x + ab + b$ | \Rightarrow
- $2f(x) + 1 = 2(ax + b) + 1 = 2ax + 2b + 1$ | \Rightarrow
- $\Rightarrow \begin{cases} a^2 - 2a = 0 \\ ab + b - 2b - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} a(a-2) = 0 \\ ab - b - 1 = 0 \end{cases}; a \neq 0 \Rightarrow$

$$\Rightarrow \left\{ \begin{array}{l} a = 1 \\ 2a - b - 1 = 0 \end{array} \right. \Rightarrow b = 1$$

Deci, $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x + 1$