

1) Să ne calculăm:  $\cos \frac{23\pi}{12} \cdot \sin \frac{\pi}{12}$

$$\cos \frac{23\pi}{12} = \cos \left( \frac{23\pi}{12} - 2\pi \right) = \cos \frac{-\pi}{12} =$$

$\sin a = -\sin(a - \pi)$	$= \cos \frac{\pi}{12}$
$\cos a = -\cos(a - \pi)$	

$\sin(-a) = -\sin a$	$\sin a \cdot \cos a = \frac{1}{2} \sin 2a$
$\cos(-a) = \cos a$	

$$\cos \frac{23\pi}{12} \cdot \sin \frac{\pi}{12} = \sin \frac{\pi}{12} \cdot \cos \frac{\pi}{12} =$$

$$= \frac{1}{2} \sin \frac{\pi}{6} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

3) Să ne rezolvă în  $(0, \pi)$ , ecuația  $\sin 3x = \sin x$

$$\sin 3x - \sin x = 0$$

$$\sin a - \sin b = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2}$$

$$2 \sin x \cdot \cos 2x = 0$$

- $\sin x = 0$ , nu are sol. pt.  $x \in (0, \pi)$
- $\cos 2x = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} \Rightarrow 2x \in \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$
- $2x \in (0, 2\pi)$
- $\Rightarrow x \in \left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\}$

5) Să ne arate că:

$$\sin^2 1^\circ + \sin^2 2^\circ + \dots + \sin^2 90^\circ = \frac{91}{2}$$

$$\sin(90^\circ - x) = \cos x$$

$$\cos(90^\circ - x) = \sin x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin 89^\circ = \sin(90^\circ - 1^\circ) = \cos 1^\circ$$

$$\Rightarrow \sin^2 1^\circ + \sin^2 89^\circ = \sin^2 1^\circ + \cos^2 1^\circ = 1$$

$$\sin 88^\circ = \sin(90^\circ - 2^\circ) = \cos 2^\circ$$

$$\Rightarrow \sin^2 2^\circ + \sin^2 88^\circ = \sin^2 2^\circ + \cos^2 2^\circ = 1$$

...

2)  $\Delta ABC$  este un triunghi.  $\angle A = 60^\circ$ ,  $b = 10$ ,  $S = 15\sqrt{3}$ ,  $a = ?$

$$S = \frac{bc \sin A}{2}$$

$$15\sqrt{3} = \frac{60}{2} \cdot \sin A \Rightarrow \sin A = \frac{\sqrt{3}}{2}$$

$$\pi \in (0, \frac{\pi}{2}) \Rightarrow A = \frac{\pi}{3}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$a^2 = 100 + 36 - 120 \cdot \frac{1}{2} \Rightarrow a^2 = 76$$

$$\Rightarrow a = 2\sqrt{19}$$

4)  $\Delta ABC$ ,  $a = 3$ ,  $b = 4$ ,  $c = 5$ ,  $r = ?$   
( $r$  = raza cercului inscris în  $\Delta ABC$ )

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

$$p = \frac{1}{2}(a+b+c)$$

$$S = p \cdot r$$

$$p = \frac{1}{2}(3+4+5) = 6$$

$$S = \sqrt{6 \cdot 3 \cdot 2 \cdot 1} = 6$$

$$r = \frac{S}{p} = 1$$

$$\sin 46^\circ = \sin(90^\circ - 44^\circ) = \cos 44^\circ \Rightarrow$$

$$\sin^2 44^\circ + \sin^2 46^\circ = \sin^2 44^\circ + \cos^2 44^\circ = 1$$

$$\begin{aligned} & \Rightarrow \sin^2 1^\circ + \sin^2 2^\circ + \dots + \sin^2 90^\circ = \\ & = \underbrace{1 + 1 + \dots + 1}_{44 de 1} + \sin^2 45^\circ + \sin^2 90^\circ = \\ & = 44 + \frac{1}{2} + 1 = \frac{91}{2} \end{aligned}$$

6)  $\triangle ABC$ ,  $A = \frac{\pi}{4}$ ,  $B = \frac{\pi}{6}$ ,  $c = 6$   
 $R = ?$ ,  $R$  = raza cercului circumscris  $\triangle ABC$ .

$$C = \pi - A - B = \pi - \frac{\pi}{4} - \frac{\pi}{6} = \frac{7\pi}{12}$$

$$\sin C = \sin \frac{7\pi}{12} = \sin \left( \frac{\pi}{4} + \frac{\pi}{3} \right) =$$

$$= \sin \frac{\pi}{4} \cos \frac{\pi}{3} + \cos \frac{\pi}{4} \sin \frac{\pi}{3} =$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\sin(a+b) = \sin a \cdot \cos b + \\ + \sin b \cdot \cos a$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$2R = \frac{c}{\sin C} \Rightarrow 2R = \frac{6}{\frac{\sqrt{2} + \sqrt{6}}{4}} \quad | :2$$

$$\Rightarrow R = \frac{12}{\sqrt{6} + \sqrt{2}} = \frac{12(\sqrt{6} - \sqrt{2})}{6 - 2} \Rightarrow$$

$$\Rightarrow R = 3(\sqrt{6} - \sqrt{2})$$

7) Să se rezolve în  $[0, 2\pi)$  ecuația

$$\cos 2x = \frac{1}{2}$$

$$\cos x = a \quad | \Rightarrow x = \pm \arccos a + k\pi, \\ a \in [-1, 1] \quad k \in \mathbb{Z}$$

$$\sin x = a \Rightarrow x = (-1)^k \arcsin a + k\pi, \\ a \in [-1, 1] \quad k \in \mathbb{Z}$$

$$\tan x = a \Rightarrow x = \arctan a + k\pi, \\ a \in \mathbb{R} \quad k \in \mathbb{Z}$$

$$\cot x = a \Rightarrow x = \operatorname{arccot} a + k\pi, \\ a \in \mathbb{R} \quad k \in \mathbb{Z}$$

$$2x = \pm \arccos \frac{1}{2} + 2k\pi, \quad k \in \mathbb{Z}$$

$$2x = \pm \frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$$

$$x = \pm \frac{\pi}{6} + k\pi, \quad k \in \mathbb{Z}$$

Dacă  $x \in [0, 2\pi)$ . Dăm valori lui  $k \in \mathbb{Z}$

$$\cdot \text{pt. } k < 0 \Rightarrow x < 0$$

$$\cdot \text{pt. } k = 0 \Rightarrow x = \pm \frac{\pi}{6} \Rightarrow x = \frac{\pi}{6} \in [0, 2\pi)$$

$$\cdot \text{pt. } k = 1 \Rightarrow x = \pm \frac{\pi}{6} + \pi$$

$$x = \frac{\pi}{6} + \pi = \frac{7\pi}{6} \in [0, 2\pi)$$

$$x = -\frac{\pi}{6} + \pi = \frac{5\pi}{6} \in [0, 2\pi)$$

$$\cdot k = 2 \Rightarrow x = \pm \frac{\pi}{6} + 2\pi$$

$$x = \frac{\pi}{6} + 2\pi = \frac{13\pi}{6} \notin [0, 2\pi)$$

$$x = -\frac{\pi}{6} + 2\pi = \boxed{\frac{11\pi}{6}} \in [0, 2\pi)$$

$$\cdot \text{pt. } k \geq 3 \Rightarrow x > 2\pi$$

$$\text{Deci } S = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

$$8) \triangle ABC, a = 5, b = 6, c = 4$$

$$\text{SSAC: } B = 2C$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{25 + 16 - 36}{40} = \frac{1}{8}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{25 + 36 - 16}{60} = \frac{3}{4}$$

$$\cos 2C = 2 \cos^2 C - 1 = 2 \cdot \frac{9}{16} - 1 = \frac{1}{8}$$

$$\Rightarrow \cos B = \cos 2C \quad (1)$$

$$\cos B > 0 \Rightarrow B \in (0, \frac{\pi}{2})$$

$$\cos C > 0 \Rightarrow C \in (0, \frac{\pi}{2}) \Rightarrow 2C \in (0, \pi)$$

$$\text{Deci } B, 2C \in (0, \pi) \quad (2)$$

$$\text{"cos" este injectivă pe } (0, \pi) \quad (3)$$

$$\Rightarrow B = 2C$$

8) Să se rezolve în  $[0, 2\pi]$  ecuația  $\sin x + \cos x = -1$

Ridicăm la patrat (va fi b. să verific oală)

$$\Rightarrow \underbrace{\sin^2 x + \cos^2 x}_{=1} + 2 \sin x \cos x = 1$$

$$\Rightarrow 2 \sin x \cos x = 0$$

- $\sin x = 0 \quad \left. \begin{array}{l} \\ x \in [0, 2\pi] \end{array} \right\} \Rightarrow x \in \{0, \pi\}$
- $\cos x = 0 \quad \left. \begin{array}{l} \\ x \in [0, 2\pi] \end{array} \right\} \Rightarrow x \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

Deci  $S = \{\pi, \frac{3\pi}{2}\}$

Verificare:

$x = 0 : \frac{\sin 0}{0} + \frac{\cos 0}{1} = -1 \Leftrightarrow 1 = -1 \text{ (F)}$

$x = \pi : \frac{\sin \pi}{0} + \frac{\cos \pi}{-1} = -1 \text{ (A)}$

$x = \frac{\pi}{2} : \frac{\sin \frac{\pi}{2}}{1} + \frac{\cos \frac{\pi}{2}}{0} = -1 \text{ (F)}$

$x = \frac{3\pi}{2} : \frac{\sin \frac{3\pi}{2}}{-1} + \frac{\cos \frac{3\pi}{2}}{0} = -1 \text{ (A)}$

$$a = b \Rightarrow a^2 = b^2, \quad a^2 = b^2 \nRightarrow a = b$$

$$a = b \Leftrightarrow a^2 = b^2$$

9) Să se rezolve în  $[0, 2\pi]$  ecuația  $\sin x + \cos x = -1$

metoda 2

$$a \sin x + b \cos x = c \quad | : \sqrt{a^2 + b^2}$$

$a = 1, b = 1, c = -1$

$$\sqrt{a^2 + b^2} = \sqrt{2}$$

împărțim ecuația prim  $\sqrt{2}$ :

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = -\frac{1}{\sqrt{2}} \Leftrightarrow$$

$$\sin x \cdot \cos \frac{\pi}{4} + \cos x \cdot \sin \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\sin(x + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$$

$$x + \frac{\pi}{4} = (-1)^k \cdot \arcsin(-\frac{\sqrt{2}}{2}) + k\pi, \quad k \in \mathbb{Z}$$

$$x + \frac{\pi}{4} = (-1)^k \cdot \left(-\frac{\pi}{4}\right) + k\pi, \quad k \in \mathbb{Z}$$

$$x = (-1)^{k+1} \cdot \frac{\pi}{4} - \frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}$$

- pt.  $k < 0 : x < 0$
- pt.  $k = 0 : x = -\frac{\pi}{2} \notin [0, 2\pi]$
- pt.  $k = 1 : x = \boxed{\pi} \in [0, 2\pi]$
- pt.  $k = 2 : x = -\frac{\pi}{2} + 2\pi = \boxed{\frac{3\pi}{2}} \in [0, 2\pi]$
- pt.  $k \geq 3 : x > 2\pi$

Deci  $S = \{\pi, \frac{3\pi}{2}\}$ .

9) Să se rezolve în  $[0, 2\pi]$  ecuația  $\sin x + \cos x = -1$

metoda 3

$$a \sin x + b \cos x = c$$

- se verifică dacă  $x = (2k+1)\pi, \quad k \in \mathbb{Z}$  este soluție ( $-b = c$ )
- pt.  $x \neq (2k+1)\pi, \quad k \in \mathbb{Z}$

Notăm  $\tan \frac{x}{2} = t$

$$\sin x = \frac{2t}{1+t^2}, \quad \dots t = \dots$$

$$\cos x = \frac{1-t^2}{1+t^2}, \quad x = \dots$$

- $x = (2k+1)\pi, \quad k \in \mathbb{Z}$
- $0 - 1 = -1 \text{ (A)}$
- $x = \pi \text{ sol.}$
- pt.  $x \neq (2k+1)\pi, \quad k \in \mathbb{Z}$
- $\tan \frac{x}{2} = t$
- $\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = -1$
- $\Rightarrow t = -1 \Rightarrow \tan \frac{x}{2} = -1$
- $\Rightarrow \frac{x}{2} = -\frac{\pi}{4} + k\pi \Rightarrow x = -\frac{\pi}{2} + 2k\pi$
- $x \in [0, 2\pi) \Rightarrow x = \boxed{\frac{3\pi}{2}}$