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Inequalities on Isotropic Submanifolds in Pseudo-Riemannian Space Forms

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Abstract

Both spacelike and isotropic submanifolds of pseudo-Riemannian spaces have interesting properties, studied in Mathematics and Physics as well. We will present new inequalities for isotropic spacelike submanifolds of pseudo-Riemannian space forms, respectively a corresponding inequality of a generalized Euler inequality and a Ricci inequality [1]. Other submanifolds will be considered.

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A study of normalized null hypersurfaces of indefinite almost Hermitian manifolds

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Abstract

In this paper, we extend the notion of rigging technique from a null hypersurface of Lorentzian manifolds to a null hypersurface of indefinite almost Hermitian manifolds. We construct an associated Hermitian metric $\breve{g}$ on an indefinite almost Hermitian manifold $(\overline{M}, \overline{g})$ with a fixed $J-$rigging ζ and derive an induced non-degenerate $J-$rigged metric $\widetilde{g}$ on its normalized null hypersurface. We prove that the structures $(\overline{g}, J)$ and $(\breve{g}, J)$ are not simultaneously Kaehlerian. We obtain equations linking the Levi-Civita connections $\overline{\nabla}$ and $\widetilde{\nabla}$ of $\overline{g}$ and $\widetilde{g}$, respectively. We also derive Gauss-Weingarten type formulae for null hypersurface M of an indefinite Kaehler manifold $(\overline{M}, \overline{g}, J)$ with a fixed closed Killing $J-$rigging ζ for M. Finally, we establish some relations linking the curvatures, sectional curvatures, holomorphic sectional curvatures, holomorphic bisectional curvatures and null sectional curvatures etc. of the ambient manifold $\overline{M}$ and normalized null hypersurface (M, ζ).

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Empirical Analysis of Reliability of Series-Parallel versus Parallel-Series Networks by Monte Carlo Methods

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Abstract

In this paper, the comparative empirical analysis of the reliability of serial-parallel networks versus parallel-serial networks for dynamic models was performed. These dynamic models were previously exposed to analytical analysis for the case when the numbers of subnets and units in each subnet are predefined, constant numbers, but also when the lifetimes of the units are independent random variables. The work contains a program product intended for Monte-Carlo simulation for calculating the reliability of related networks. The estimators corresponding to the model, the required quantiles and the sample sizes corresponding to the simulation were calculated. With the help of these simulations, deduced functions were validated for the dynamic model, which is less studied and proved to be relevant for the static model as well. All the simulation results were collected in tables and displayed graphically for the convenience of analyzing the results. The simulations were done for two types of network structures, using a few examples for which analytical formulas are known, but can be extended to a wider range of more complex network models.

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Generalized Normal Ruled Surfaces of Frenet-Type Framed Curves

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Abstract

In this study, we introduce generalized normal ruled surfaces of Frenet-type framed curves with singular points. We investigate the geometrical properties and structures of these surfaces. Also, by using the basic singularity theory framed curves, we give singularity types of generalized normal surfaces. Moreover, since these surfaces have singular points, we examine them as framed surfaces we calculate the basic invariants of the surface.

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Matrix representations of quaternion algebras over real numbers and over complex numbers

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Abstract

In this paper I will develop two large chapters related to quaternion algebras over two known sets. In the first chapter, I will study the properties and matrix representations of quaternion algebras over the set of complex numbers, and in the second one, over the set of real numbers..

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Approximation properties of some nonpositive Kantorovich type operators

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Abstract

In this paper we will construct a generalization of Bernstein operators using Kantorovich's method. In this sense we will use a general derivative operator denoted by D^l and its corresponding anti-derivative operator I^l, having the property $D^l \circ I^l = I^l \circ D^l = Id$. We will prove that the convergence on all continuous functions on $[0,1]$ holds even though the operators constructed this way are not positive. Also, a Voronovskaja type theorem will be proved for this class of operators.

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An eigenvalue problem involving the (p,q)-Laplacian with a parametric boundary condition

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Abstract

Let Ω in $\mathbb{R}^N$, $N \geq 2$, be a bounded domain with smooth boundary $\partial\Omega$. Consider the following nonlinear eigenvalue problem

$$\begin{cases} -\Delta_p u - \Delta_q u + \rho(x) \mid u \mid^{q-2} u = \lambda\alpha(x) \mid u \mid^{r-2} u & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu_{pq}} + \gamma(x) \mid u \mid^{q-2} u = \lambda\beta(x) \mid u \mid^{r-2} u & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $p, q, r \in (1, \infty)$ with $p \neq q$, $\alpha, \rho \in L^\infty(\Omega)$, $\beta, \gamma \in L^\infty(\partial\Omega)$, $\Delta_\theta u :=$ div $(\|\nabla u\|^{\theta-2}\nabla u)$, $\theta \in \{p, q\}$, and $\frac{\partial u}{\partial \nu_{pq}}$ denotes the conormal derivative corresponding to the differential operator $-\Delta_p - \Delta_q$. Under suitable assumptions we provide the full description of the spectrum of the above problem in eight cases out of ten, and for other two complementary cases we obtain subsets of the corresponding spectra. Notice that when some of the potentials $\alpha, \beta, \rho, \gamma$ are null functions, the above eigenvalue problem reduces to Neumann, Robin or Steklov type problems, and so we obtain the spectra of these particular eigenvalue problems.

The operator $\big(\Delta_p + \Delta_q\big)$, called (p, q)-Laplacian, occurs in many two phase models arising in mathematical physics. For example, if $p = 2$ and $q > 1$, the operator $\big(\Delta + a_q\Delta_q\big)$, $a_q > 0$ has applications in Born-Infeld theory for electrostatic fields (see Bonheure, Colasuonno and Fortunato [4], Fortunato, Orsina and Pisani [6]). The presence of the positive constant a_q instead of 1 is not important. We also refer to Benci et al. [2] and Benci, Fortunato and Pisani [3] for more general applications to quantum physics. Two phase equations arise also in other parts of mathematical physics as reaction diffusion systems (see Cherfils and Il'yasov [5]) and nonlinear elasticity theory (see Marcellini [10] and Zhikov [12]).

The study of eigenvalue problems with such a boundary condition is motivated by the particular case $p = q = 2$, $\alpha \equiv 1$, $\beta \equiv$ const. > 0 which was investigated by Von Below and François in [11] (see also [7]). They call it a dynamical eigenvalue problem as it can be derived from the study of a parabolic problem with dynamical boundary conditions. Similarly, the motivation behind problem (1) comes from the study of a double phase parabolic equation (see Arora and Shmarev [1], Huang [8], Marcellini [9] and the references therein) under a dynamical boundary condition. The existence theory for local solutions of such parabolic problems relies on the spectral theory of the associated elliptic problem with the eigenvalue parameter both in the equation and the boundary condition.

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An integral type fixed point theorem

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Abstract

In the present paper we prove an integral type metrical fixed point theorem for non-self mappings. The existence of fixed point is ensure by hypotheses formulated in terms of equivalent metric spaces. Some illustrative examples are also furnished to support the main result.

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Computing the fuzzy grade of hypergroups of small cardinality

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Abstract

The theory of hyperstructures started in the 1930's thanks to Marty ([6]) and still many unexplored paths exist in it and are studied to this day ([5]).

In 2003 Piergiulio Corsini introduced ([2]) the concept of fuzzy grade for a hypergroupoid thus increasing the amount of connections between hypergroup theory and fuzzy set theory.

The first part of the talk is dedicated to introducing the concept of hypergroupoid and in particular the complete hypergroups studied in detail by Corsini, Angheluţă and Cristea ([1], [4]).

The concept of join space will then be explained, including the different approaches one can have regarding it (either building it starting from a hypergroup or building it with geometry axioms).

The last ideas that are needed to build the foundations required in the follow up are the ones of fuzzy set (see Zadeh, [7]) and fuzzy grade (that was first given this name in [3]) that will be discussed in the third part.
Once everything is settled it's possible to start investigating some properties of the fuzzy grade, in particular one can notice how in some special cases we can avoid computations, either because we know that two hypergroupoids are isomorphic or because we already know the fuzzy grade (the main example of such an occurrence are hypergroups of cardinality 2^g where every element has a different image under the fuzzy set).

The previous work regarding complete hypergroups will then be reviewed.

Finally an algorithm in sagemath will be introduced that can be used to compute the fuzzy grade of the join spaces of the type found in the construction. This is particularly useful in order to look for patterns and check patterns we suppose exists, all with the goal of finding more hypergroups whose fuzzy grade can be get without running through all the computations.

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On conformal Riemannian maps whose total manifold admits a Ricci soliton

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Abstract

We study conformal Riemannian maps between the Riemannian manifolds. We derive conditions for such maps to be harmonic. Later, we study conformal Riemannian maps whose total manifold admits a Ricci soliton and present a non-trivial example of such conformal Riemannian maps. We also obtain conditions for fiber and range space of such maps to be Ricci soliton and Einstein. We derive conditions for conformal Riemannian maps whose total manifold admits a Ricci soliton to be harmonic and biharmonic.

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Majorization results for h_d-uniformly Schur convex functions and ω-m-star convex functions

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Abstract

We present some extensions of majorization results into the framework of h_d-uniformly Schur convex functions. More precisely, we consider the case of functions which still remain Schur-convex after substracting an homogeneous symmetric polynomials of even degree. Moreover, we deal with ω-m-star convex functions into the case of ordered Banach spaces, where the Hardy-Littlewood-Polya inequality of majorization is still holds. [1]

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Application of the distribution of the Jones-Larsen order statistics in the study of city size

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Abstract

In this presentation we analyze the weak stochastic order between Jones–Larsen distributions and we present an application in the study of the size of cities in Spain and Romania between 1998–2007. Finally, we will discuss the importance using this distribution model.

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A new generalized Bernstein operator

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Abstract

In this paper we investigate the properties of a new generalized Bernstein type operators.

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Curvature invariants of Lagrangian submanifolds in quaternionic space forms

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Abstract

Using an optimization technique on Riemannian submanifolds, we prove some sharp inequalities for δ-Casorati curvature invariants of Lagrangian submanifolds in quaternionic space forms, i.e. quaternionic Kähler manifolds of constant q-sectional curvature. We show that in the class of Lagrangian submanifolds in quaternionic space forms, there are only two subclasses of ideal Casorati submanifolds, namely the family of totally geodesic submanifolds and a particular subfamily of H-umbilical submanifolds. Finally, we provide some examples to illustrate the obtained results. In particular, we point out that an entire family of ideal Casorati Lagrangian submanifolds can be constructed using the concept of quaternionic extensor introduced by Oh and Kang in [1, 2]. This is a joint work with M. Aquib, M. S. Lone and G.-E. Vîlcu.

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Some results regarding the levels and sublevels of quaternion algebras

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Abstract

The study of the level and sublevel of quaternion algebras is closely related to bilinear forms attached to these algebras. Knowing the level of a field is as important as knowing the characteristic of that field.

The level of a field is the smallest natural number n such that -1 can be expressed as a sum of n squares. Otherwise, if -1 cannot be expressed as a sum of squares, then we define the level as infinite. The same definition is kept for commutative rings.

A famous result by Pfister states that if a field has a finite level, then this level must be a power of 2, and later Dai, Lam and Peng proved that any positive integer can be the level of a commutative ring.

We present the usual notion of level for quaternion division algebra and some properties. There are known examples of quaternion division algebra of level 2^k and $2^k + 1$, for any $k \geq 0$, constructed by D.W. Lewis, but till now, it is not known exactly what integer numbers can be considered as the level of a quaternion algebra or if there exist quaternion division algebras whose levels are not of this form.

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Conformal slant submersions from nearly Kaehler manifold

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Abstract

We study slant submersions and conformal slant submersions from nearly Kaehler manifolds onto Riemannian manifolds and investigate conditions for such maps to be totally geodesic maps. We also obtain conditions for a slant submersion and a conformal slant submersion from a nearly Kaehler manifold onto a Riemannian manifold to be a harmonic map and a harmonic morphism, respectively.

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Almost Norden submersions between almost Norden manifolds

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Abstract

We define an almost Norden submersion (holomorphic and semi-Riemannian submersion) between almost Norden manifolds and show that, in most of the cases, the base manifold has the similar kind as that of total manifold. We obtain necessary and sufficient conditions for almost Norden submersion to be a totally geodesic map. We also derive decomposition theorems for the total manifold of such submersions. Moreover, we study the harmonicity of almost Norden submersions between almost Norden manifolds and between Kaehler-Norden manifolds. Finally, we derive conditions for an almost Norden submersion to be a harmonic morphism.

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Weak Convergence Theorems for Inertial Krasnoselskii-Mann Iterations in the class of enriched nonexpansive operators in Hilbert Spaces

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Abstract

In this paper, we present some results about the aproximation of fixed points of nonexpansive and enriched nonexpansive operators. There are numerous works in this regard (for example [6], [7], [9], [10], [14], [16], [35] and references to them). Of course, the bibliografical references are extensive and they are mentioned at the end of this paper. In order to approximate the fixed points of enriched nonexpansive mappings, we use the Krasnoselskii-Mann iteration for which we prove weak convergence theorem and the theorem which offers the convergence rate analysis.
Our results in this paper extend some classical convergence theorems from the literature from the case of nonexpansive mappings to that of enriched nonexpansive mappings. One of our contributions is that the convergence analysis and rate of convergence results are obtained using conditions which appear not complicated and restrictive as assumed in other previous related results in the literature.

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A Voronovskaya type theorem associated to geometric series of Bernstein - Durrmeyer operators

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Abstract

In this paper we will introduce a quantitative estimate of the convergence of the operators introduced by U. Abel, that is, we will obtain a Voronovskaya type theorem concerning some operators which are defined as the geometric series of Bernstein - Durrmeyer operators, on a certain subspace of L^∞ integrable functions on an interval I. Also, regarding the operators we mentioned, we will obtain an identity which will be used to prove our main result.

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The Induced Representations of the Group G

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Abstract

We describe the semi-direct product of the Heisenberg group and the a ne group which is called the group G: The group G is a subgroup of the Schrodinger group. We study the group G and its Lie algebra. Then, we find the induced representations of the group G.

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On an elliptic-like regularization of semilinear evolution equations in Hilbert spaces

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Abstract

In a Hilbert space H, consider the Cauchy problem

$$\begin{cases} u'(t) + Au(t) + Bu(t) = f(t), \quad t \in [0,T], \\ u(0) = u_0, \end{cases} \tag{P_0}$$

where $T > 0$ is a given time instant, $u_0 \in H$ is a given initial state, $f : [0,T] \to H$ and
(h_A) $A : D(A) \subset H \to H$ is a linear, maximal monotone operator;
(h_B) $B : H \to H$ is a nonlinear Lipschitz operator on H, i.e., there exists $L > 0$ such that $\forall x, y \in H, \| Bx - By \| \leq L \| x - y \|_H$.

Following the method of artificial viscosity introduced by J.L. Lions [7], we associate with problem (P_0) the approximate problem (P_ε) :

$$\begin{cases} -\varepsilon u''(t) + u'(t) + Au(t) + Bu(t) = f(t), \quad t \in [0,T], \\ u(0) = u_0, \quad u'(T) = 0, \end{cases} \tag{P_ε}$$

where ε is a positive small parameter. We investigate existence, uniqueness and higher regularity for the solutions of problems (P_0) and (P_ε). Then we establish asymptotic expansions of order zero and of order one for the solution of problem (P_ε). Problem (P_ε) turns out to be regularly perturbed of order zero and singularly perturbed of order one with respect to the norm of $C([0,T];H)$. However, the boundary layer of order one is not visible through the norm of $L^2(0,T;H)$. This paper is a significant extension of a previous one by M. Ahsan and G. MoroÅŸanu [2]. The framework created here allows the treatment of hyperbolic problems (besides parabolic ones). Specifically, our main result is illustrated with the semilinear telegraph system (thus extending a result by N.C. Apreutesei and B. Djafari Rouhani [3]) and the semilinear wave equation.

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Congruences with $q-$ generalized Catalan numbers and $q-$harmonic numbers

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Abstract

In this paper, we give some congruences related to $q-$ generalized Catalan numbers, $q-$harmonic numbers and alternating $q-$harmonic numbers.

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Positional Adapted Frame in the Three-Dimensional Lie Groups and a Berry Phase Model

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Abstract

In this study, we introduce Positional Adapted Frame (PAF) in the three-dimensional Lie groups. Also, we give Frenet-Serret type derivative formulas with respect to PAF by means of Lie curvature and examine some special cases of Lie curvature in the three-dimensional Lie groups. Then, we construct Berry phase model of the polarized light wave along an optical fiber related to PAF in the three-dimensional Lie groups.

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